

Formal Verification and Specification
Lab Session (TP) 04

1. LTL equivalence:

- Express $\Box p$ with the operators $\neg$, $\Diamond$ and p .
- Express $\Diamond p$ with the operator U .
- Can we express $\bigcirc$ using other operators ?
- Can we express U using other operators?

2. LTL Buchi automaton:

Transform the following LTL formulae into Buchi automaton :

$p, \neg p, Xp, Fp, Gp, pUq, FGa$.

3. LTL Buchi automaton:

Express in LTL the following properties:

- in the future state, if p is true, q is never true
- p will be true one time at most
- p will be true Exactly two times

4. LTL Buchi product:

Consider the following transition system TS on $AP = a, b$:

and the following LTL formula: $\varphi = \Box \Diamond \neg a$

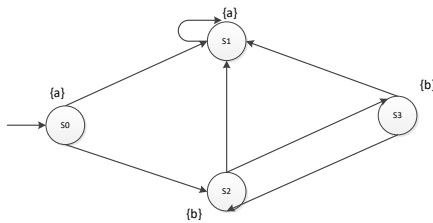


Figure 1: TS

- Derive a buchi automaton A for the formula $\neg \varphi$
- Construct the product $TS \otimes A$
- Does the transition system satisfy the formula φ ($TS \models \varphi$)
- Provide a counterexample (accepting path) if φ does not hold.

Answers

1. LTL equivalence:

- Express $\Box p$ with the operators $\neg$, $\Diamond$ and p .
- Express $\Diamond p$ with the operator U .
- Can we express $\bigcirc$ using other operators ?
- Can we express U using other operators?

Answer: • $\Box p \equiv \neg F \neg p$

- $\Diamond p \equiv true U p$
- NO, the other connectors are looser. $\bigcirc$ is exact
- No, U is mainly used between two subformulas

2. LTL Buchi automaton:

Transform the following LTL formulae into Buchi automaton :
 p , $\neg p$, Xp , Fp , Gp , pUq , FGa .

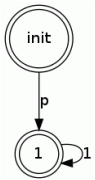


Figure 2: p

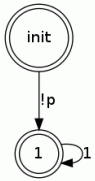


Figure 3: p

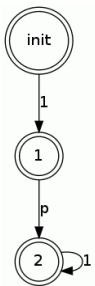


Figure 4: $X p$

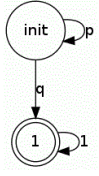


Figure 7: $p \cup q$

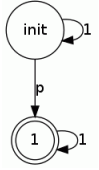


Figure 5: $F p$



Figure 6: $G p$

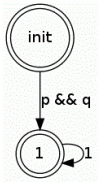


Figure 8: $p \wedge q$

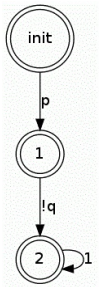


Figure 9: $p \wedge X \neg q$

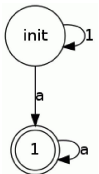


Figure 10: FGa

Answer:

3. LTL Buchi automaton:

Express in LTL the following properties:

- in the future state, if p is true, q is never true
- p will be true one time at most
- p will be true Exactly two times

Answer: • $X(p \implies G\neg q)$ or $((Xp) \implies (XG\neg q))$

- $((G\neg p) \vee (\neg pU(p \wedge X(G\neg p))))$
- $(\neg pU(p \wedge X(\neg pU(p \wedge X(G\neg p)))))$

4. LTL Buchi product:

Consider the following transition system TS on $AP = a, b$:
and the following LTL formula: $\varphi = \Box\Diamond\neg a$

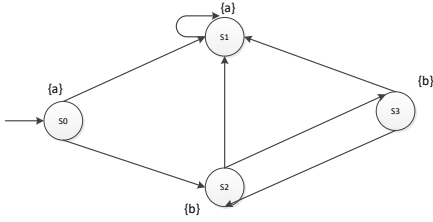


Figure 11: TS

- Derive a buchi automaton A for the formula $\neg\varphi$
- Construct the product $TS \otimes A$
- Does the transition system satisfy the formula φ ($TS \models \varphi$)
- Provide a counterexample (accepting path) if φ does not hold.

Answer: $\neg\varphi \equiv \neg\Box\Diamond\neg a \equiv \Diamond\Box a$ The automaton A for $\Diamond\Box a$ is the following:

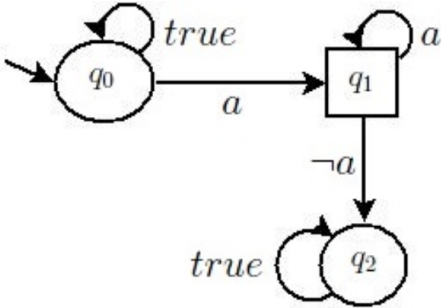


Figure 12: TS

The product $TS \otimes A$ is :

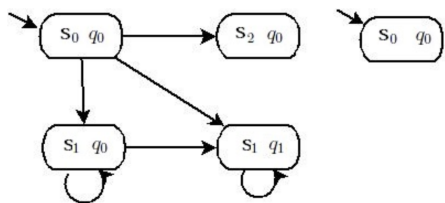


Figure 13: Product

$TS \not\models A$ and a counterexample is the path $\pi = s0(s1)^\omega$