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Formal Specification and Verification

Chapter 02: Logical Notations and Set Theory basics

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Logical notation

Basic

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Symbol	Signification	Definition
$\wedge$	logical and	
$\neg$	negation	
$\vee$	logical or	$a \vee b \stackrel{def}{=} \neg(\neg a \wedge \neg b)$
$\Rightarrow$	implication	$a \Rightarrow b \stackrel{def}{=} \neg a \vee b$
$\leftrightarrow$	equivalence	$a \leftrightarrow b \stackrel{def}{=} a \Rightarrow b \wedge b \Rightarrow a$

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Symbole	Signification	Syntaxe	Définition
$\forall$	pour tout	$\forall Id_liste \cdot (Prédicat)$	$\exists x \cdot (P) \stackrel{def}{=} \neg \forall x \cdot (\neg P)$
$\exists$	il existe	$\exists Id_liste \cdot (Prédicat)$	

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Symbole	Signification	Définition
$\in$	appartient à	
$\notin$	n'appartient pas à	$x \notin s \stackrel{def}{=} \neg (x \in s)$
$\subseteq$	est inclus dans	$s \subseteq t \stackrel{def}{=} s \in \mathbb{P}(t)$
$\not\subseteq$	n'est pas inclus dans	$s \not\subseteq t \stackrel{def}{=} \neg (s \subseteq t)$
$\subset$	est strictement inclus dans	$s \subset t \stackrel{def}{=} (s \subseteq t \wedge s \neq t)$
$\not\subset$	n'est pas strictement inclus dans	$s \not\subset t \stackrel{def}{=} \neg (s \subset t)$

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	Signification	Définition
$\cup$	union	$s_1 \cup s_2 \stackrel{def}{=} \{x \mid x \in t \wedge (x \in s_1 \vee x \in s_2)\}$
$\cap$	intersection	$s_1 \cap s_2 \stackrel{def}{=} \{x \mid x \in t \wedge (x \in s_1 \wedge x \in s_2)\}$
$-$	différence d'ensembles	$s_1 - s_2 \stackrel{def}{=} \{x \mid x \in t \wedge (x \in s_1 \wedge x \notin s_2)\}$

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$$s \subseteq s$$

(réflexivité)

$$s \subseteq t \wedge t \subseteq u \Rightarrow s \subseteq u$$

(transitivité)

$$s \subseteq t \wedge t \subseteq s \Rightarrow s = t$$

(anti-symétrie)

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- they represent a set of couples
- Relations are useful in specifying invariants, properties...

Symbole	Signification	Définition
$\leftrightarrow$	relation entre deux ensembles	$E_1 \leftrightarrow E_2 \stackrel{def}{=} \mathbb{P}(E_1 \times E_2)$

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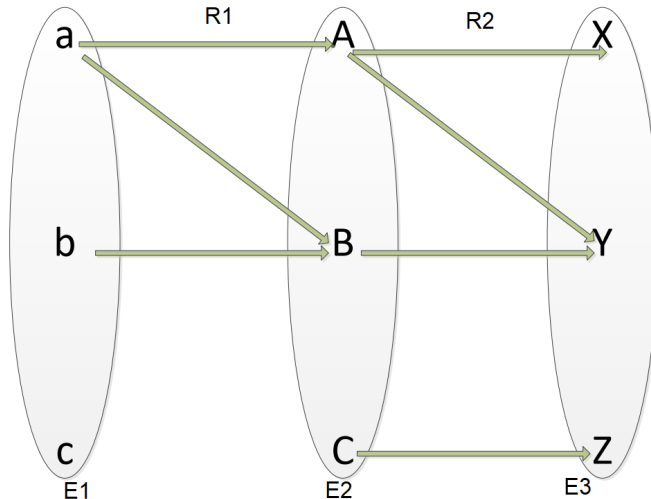
Condition	Expression	Définition
$r \in E_1 \leftrightarrow E_2$	$\text{dom}(r)$	$\{x \mid x \in E_1 \wedge \exists y \cdot (y \in E_2 \wedge (x \mapsto y) \in r)\}$
$r \in E_1 \leftrightarrow E_2$	$\text{ran}(r)$	$\{y \mid y \in E_2 \wedge \exists x \cdot (x \in E_1 \wedge (x \mapsto y) \in r)\}$
$r \in E_1 \leftrightarrow E_2$ et $F \subseteq E_1$	$r[F]$	$\{y \mid y \in E_2 \wedge \exists x \cdot (x \in F \wedge (x \mapsto y) \in r)\}$

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- $R1 : E1 \leftrightarrow E2 \quad R1 = \{(a, A), (a, B), (b, B)\} = \{a \mapsto A, a \mapsto B, b \mapsto B\}$
- $R2 : E2 \leftrightarrow E3 \quad R2 = \{(A, x), (A, y), (B, y), (C, z)\}$
- $dom(R1) = \{a, b\} \quad ran(R1) = \{A, B\} \quad codomain(R1) = \{A, B, C\} \quad R1[b, c] = \{B\}$
- $R1; R2 = \{(a, x), (a, y), (b, y)\}$
- $R1^{-1} = \{(A, a), (B, a), (B, b)\}$

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Condition	Expression	Définition
	$\text{id}(E)$	$\{x, y \mid (x \mapsto y) \in E \times E \wedge x = y\}$
$r \in s \leftrightarrow t$	r^{-1}	$\{y, x \mid (y \mapsto x) \in t \times s \wedge (x \mapsto y) \in r\}$
$r_1 \in t \leftrightarrow u$ et $r_2 \in u \leftrightarrow v$	$r_1 ; r_2$	$\{x, z \mid x, z \in t \times v \wedge$ $\exists y \cdot (y \in u \wedge (x \mapsto y) \in r_1 \wedge (y \mapsto z) \in r_2)\}$
$r_1 \in t \leftrightarrow u$ et $r_2 \in t \leftrightarrow v$	$r_1 \otimes r_2$	$\{x, (y, z) \mid x, (y, z) \in t \times (u \times v) \wedge$ $(x \mapsto y) \in r_1 \wedge (x \mapsto z) \in r_2\}$
$r_1 \in t \leftrightarrow u$ et $r_2 \in v \leftrightarrow w$	$r_1 \parallel r_2$	$\{(x, z), (y, a) \mid (x, z), (y, a) \in (t \times v) \times (u \times w) \wedge$ $(x \mapsto y) \in r_1 \wedge (z \mapsto a) \in r_2\}$
	$\text{prj}_1(E, F)$	$\{x, y, z \mid x, y, z \in E \times F \times E \wedge z = x\}$
	$\text{prj}_2(E, F)$	$\{x, y, z \mid x, y, z \in E \times F \times F \wedge z = y\}$

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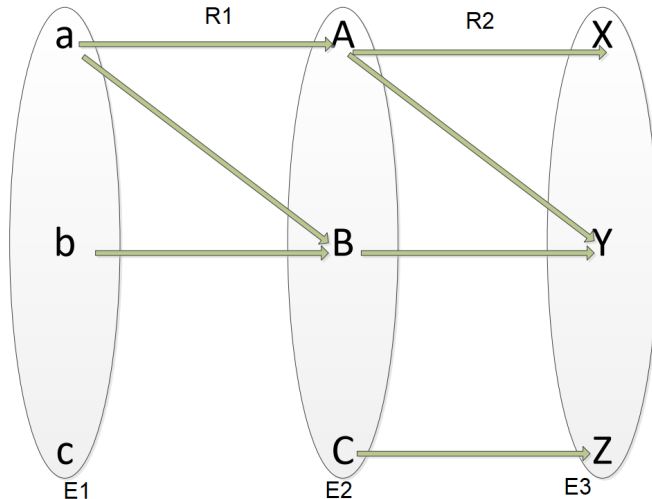
Condition	Expression	Définition
$r \in E_1 \leftrightarrow E_2$	$\text{dom}(r)$	$\{x \mid x \in E_1 \wedge \exists y \cdot (y \in E_2 \wedge (x \mapsto y) \in r)\}$
$r \in E_1 \leftrightarrow E_2$	$\text{ran}(r)$	$\{y \mid y \in E_2 \wedge \exists x \cdot (x \in E_1 \wedge (x \mapsto y) \in r)\}$
$r \in E_1 \leftrightarrow E_2$ et $F \subseteq E_1$	$r[F]$	$\{y \mid y \in E_2 \wedge \exists x \cdot (x \in F \wedge (x \mapsto y) \in r)\}$

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- $E \triangleleft R$ Restriction of R to domain E
Example: $\{b, c\} \triangleleft R1 = \{b \mapsto B\}$
- $E \triangleright R$ Restriction of R to co-domain E
Example: $R1 \triangleright \{B, C\} = \{a \mapsto B, b \mapsto B\}$
- $E \triangleleft \triangleleft R$ Anti-restriction of R to domain E
Example: $\{b, c\} \triangleleft \triangleleft R1 = \{a \mapsto A, a \mapsto B\}$
- $E \triangleright \triangleright R$ Anti-restriction of R to co-domain E
Example: $\{B, C\} \triangleright \triangleright R1 = \{a \mapsto A\}$

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- Functions are relations, in which each element of the domain is associated to one and only one element of the co-domain.
- Let A and B be two sets. A function F from A to B has the following notation: $F \rightarrow B$
- F affect exactly one element of B , denoted $F(a) \in B$, to $a \in A$, and this for every $a \in A$.
- A is called the domain, and B is the co-domain.
- b is the image of a by F
- a is the pre-image of b

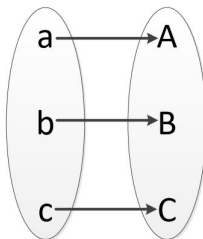


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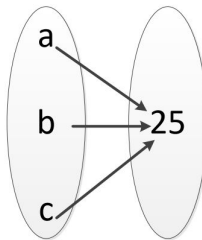
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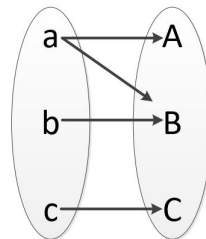
References



One to one



Many to one



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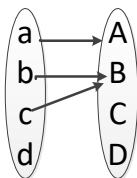
Signification	Expression	Définition
fonctions partielles	$s \leftrightarrow t$	$\{r \mid r \in s \leftrightarrow t \wedge \forall x, y, z. (x, y \in r \wedge x, z \in r \Rightarrow y = z)\}$
fonctions totales	$s \rightarrow t$	$\{f \mid f \in s \leftrightarrow t \wedge \text{dom}(f) = s\}$
injectives partielles	$s \nrightarrow t$	$\{f \mid f \in s \leftrightarrow t \wedge f^{-1} \in t \leftrightarrow s\}$
injectives totales	$s \rightharpoonup t$	$s \nrightarrow t \cap s \rightarrow t$
surjectives partielles	$s \nleftrightarrow t$	$\{f \mid f \in s \leftrightarrow t \wedge \text{ran}(f) = t\}$
surjectives totales	$s \twoheadrightarrow t$	$s \nleftrightarrow t \cap s \rightarrow t$
bijectives partielles	$s \nleftrightarrow t$	$s \nrightarrow t \cap s \nleftrightarrow t$
bijectives totales	$s \rightharpoonup t$	$s \twoheadrightarrow t \cap s \twoheadrightarrow t$

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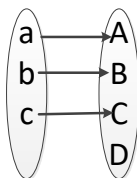
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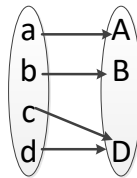
References



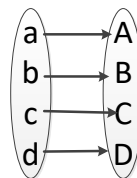
General
function



Injective
non
surjective



Surjective
non
injective



Bijjective(inj
ective and
surjective)

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- **Injective:** Every member of A has its unique matching member in B
- **Surjective:** Every member of B has at least one matching member in A or The range of function is equal to co-domain.
- **Bijective:** Every member of B has exactly one matching member in A



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References

- The B-Book: Assigning Programs to Meanings, J. R. Abrial
- Specification en B Support de cours Ecole des Jeunes Chercheurs en Programmation EJCP 2007

