University of M'sila 2024/2025
Faculty of mthematics and computer science 1^{st}year Master
Dpartement of computer science Semester 02

Exercise series N^05

Exercice 0.1. Consider the Markov chain with three states, $S = \{1, 2, 3\}$, that has the following transition matrix:

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}.$$

Draw the state transition diagram for this chain.

If we know $P(X_1 = 1) = P(X_1 = 2) = \frac{1}{4}$, find $P(X_1 = 3, X_2 = 2, X_3 = 1)$.

Exercice 0.2. Consider the Markov chain in Figure 1 There are two recurrent classes, $R_1 = \{1, 2\}$ and $R_2 = \{5, 6, 7\}$. Assuming $X_0 = 3$, find the probability that the chain gets absorbed in R_1.

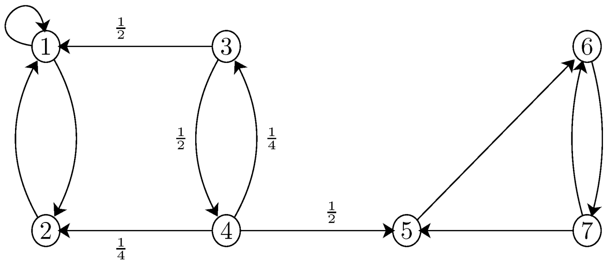

Figure 1: A state transition diagram.

Exercice 0.3. Consider the Markov chain of Example 2. Again assume $X_0 = 3$. We would like to find the expected time (number of steps) until the chain gets absorbed in R_1 or R_2. More specifically, let T be the absorption time, i.e., the first time the chain visits a state in R_1 or R_2. We would like to find $\mathbb{E}[T \mid X_0 = 3]$.

Exercice 0.4. Consider the Markov chain shown in Figure 11.19. Assume $X_0 = 1$, and let R be the first time that the chain returns to state 1, i.e.,

$$R = \min\{n \geq 1 : X_n = 1\}.$$

Find $\mathbb{E}[R \mid X_0 = 1]$.

Exercice 0.5. Consider the Markov chain shown in Figure 11.20.

Questions

1. Is this chain irreducible?
2. Is this chain aperiodic?
3. Find the stationary distribution for this chain.
4. Is the stationary distribution a limiting distribution for the chain?

Exercice 0.6. Consider the Markov chain shown in Figure 11.21. Assume that $\frac{1}{2} < p < 1$. Does this chain have a limiting distribution? For all $i, j \in \{0, 1, 2, \ldots\}$, find

$$\lim_{n \to \infty} P(X_n = j \mid X_0 = i).$$

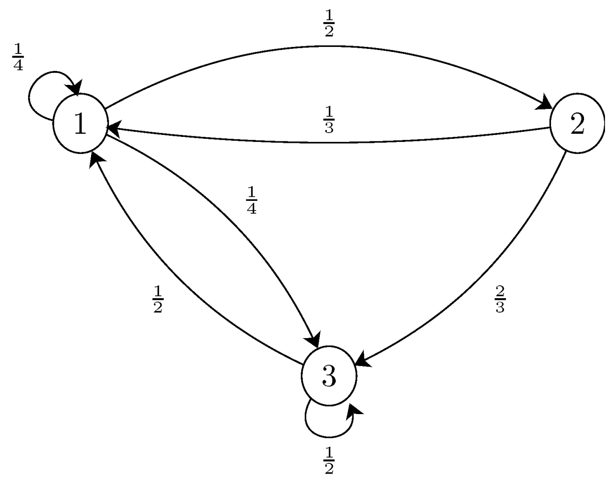

Figure 2: A state transition diagram.

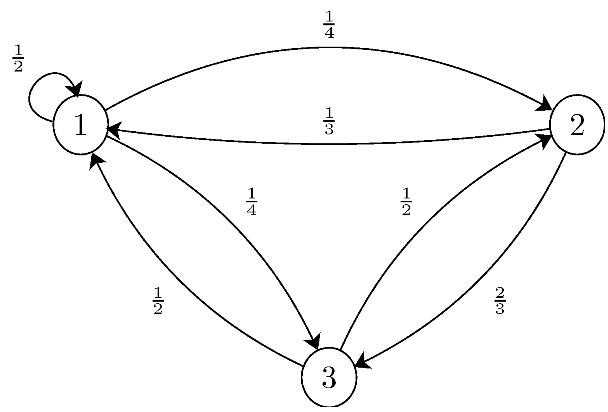

Figure 3: A state transition diagram.

Figure 4: A state transition diagram.

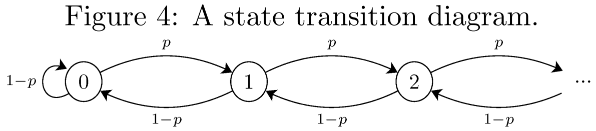

Figure 5: A state transition diagram.