University of M'sila
Faculty of mthematics and computer science
Dpartement of computer science

2024/2025
1^{st}year Master
Semester 02

Exercise series $N^0 5$

Exercice 0.1. Let U = (1,-2,4) and $A = \begin{pmatrix} 1 & 3 & -1 \\ 0 & 2 & 5 \\ 4 & 1 & 6 \end{pmatrix}$. Find uA.

Exercice 0.2. Let $A = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 & -4 \\ 3 & -2 & 6 \end{pmatrix}$ Find:

1. AB
2. BA

Exercice 0.3. Let $A = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$ Find:

1. A^2
2. A^3

Exercice 0.4. Which vectors are probability vectors?

(i) $U = (\frac{1}{8}, 0, \frac{-1}{6}, \frac{1}{2}, \frac{1}{3})$

(ii) $v = (\frac{1}{6}, 0, \frac{1}{6}, \frac{1}{2}, \frac{1}{8})$

(iii) $w = (\frac{1}{3}, 0, 0, \frac{1}{6}, \frac{1}{2})$

Exercice 0.5. Multiply each vector by the appropriate scalar to form a probability vector:

(i) $(2, 1, 0, 2, 3)$

(ii) $(4, 0, 1, 2, 0.5)$

(iii) $(3, 0, -2, 1)$

(iv) $(0, 0, 0, 0, 0)$

Exercice 0.6. Find a multiple of each vector which is a probability vector:

(i) $(\frac{1}{2}, \frac{2}{3}, 0, 2, \frac{5}{6})$

(ii) $(0, \frac{2}{3}, 1, \frac{3}{5}, \frac{5}{6})$

Exercice 0.7. Which of the following matrices are stochastic matrices ?
$A = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$, $B = \begin{pmatrix} \frac{15}{16} & \frac{1}{16} \\ \frac{2}{3} & \frac{2}{3} \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ and $D = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$

Exercice 0.8. Let
$$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$
be a stochastic matrix, and let $U = (u_1, u_2, u_3)$ be a probability vector.

- Show that UA is also a probability vector.
- If $A = (a_{ij})$ is a stochastic matrix of order n, and $U = (u_1, u_2, \ldots, u_n)$ is a probability vector, then UA is also a probability vector.

Exercice 0.9. Find the unique fixed probability vector of the regular stochastic matrix $A = \begin{pmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. What matrix does A^n approach as $n \to \infty$?

Exercice 0.10. (i) Show that the vector $\mathbf{U} = (b, a)$ is a fixed point of the general 2×2 stochastic matrix $P = \begin{pmatrix} 1-a & a \\ b & 1-b \end{pmatrix}$. . (ii) Use the result of (i) to find the unique fixed probability vector of each of the following matrices: $A = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}$ and $C = \begin{pmatrix} 0.7 & 0.3 \\ 0.8 & 0.2 \end{pmatrix}$

Exercice 0.11. Find the unique fixed probability vector of the regular stochastic matrix:
$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix}$$
. .

Exercice 0.12. Find the unique fixed probability vector of the regular stochastic matrix:
$$P = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{2} \end{pmatrix}$$
. . What matrix does P^n approach?

Exercice 0.13. Which of the following stochastic matrices are regular?
$A = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $C = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$ and $D = \begin{pmatrix} 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & 0 \end{pmatrix}$
