

Exercise series N^03

Exercise 0.1. Let X and Y be two random variables with the following joint distribution:

$X \backslash Y$	0	1
-1	$\frac{1}{8}$	$\frac{1}{8}$
0	$\frac{3}{16}$	$\frac{5}{16}$
1	$\frac{1}{8}$	$\frac{1}{8}$

- Find the marginal probability mass functions of X and Y . Are X and Y independent?
- Find the joint distribution function $F(x, y)$ at $(0, 0)$ and $(\frac{1}{2}, \frac{1}{2})$:
- Find the m.p.f. of X conditional on $Y = 1$:
- Find the expectation and variance of X , Y , and $X + Y$:

Exercise 0.2. Let X and Y be two discrete random variables both taking values in $\{0, 1, 2\}$. Suppose that their joint probability mass function is:

$$p_{XY}(x, y) = \begin{cases} 0.1 & \text{if } (x, y) \in \{(0, 0), (1, 1), (1, 2), (2, 0)\} \\ 0.2 & \text{if } (x, y) \in \{(0, 1), (0, 2), (2, 1)\} \\ 0 & \text{otherwise} \end{cases}$$

- Find the marginal probability mass functions of X and Y .
- Are the two random variables X and Y independent?
- Compute $P((X = 1) \cup (Y = 1))$ and $P((X = 1) \cap (Y = 1))$.
- Compute $P(X < 1 \mid Y > 1)$ and the expectation of X given $Y = 2$.

Exercise 0.3. Let X and Y be two discrete random variables with joint probability mass function represented in the table below:

$X \backslash Y$	0	1
-1	p	0
0	p	0
1	$\frac{1}{6}$	$\frac{1}{6}$

- Compute p :
- Find the marginal probability mass functions of X and Y . Are X and Y independent?
- Compute the expectation of X :
- Compute the expectation and the variance of Y :
- Compute the expectation of Y given $X = 0$:
- Compute $P(U = 0)$ where $U = X \cdot Y$: