

Exercises Series N^02

Exercice 0.1. Let W be a random variable giving the number of heads minus the number of tails in three tosses of a coin. List the elements of the sample space S for the three tosses of the coin and to each sample point assign a value w of W .

Exercice 0.2. Determine the value c so that each of the following functions can serve as a probability distribution of the discrete random variable X :

1. $f(x) = c(x^2 + 4)$, for $x = 0, 1, 2, 3$;
 2. $f(x) = cC_x^2 C_{3-x}^3$
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Exercice 0.3. Let X be a discrete random variable with the following PMF

$$p_X(x) = \begin{cases} 0.1 & \text{for } x = 0.2 \\ 0.2 & \text{for } x = 0.4 \\ 0.2 & \text{for } x = 0.5 \\ 0.3 & \text{for } x = 0.8 \\ 0.2 & \text{for } x = 1 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

1. Find the sample space S of the random variable X
 2. Find $p(X \leq 0.5)$
 3. Find $0.25 < p(X \leq 0.75)$
 4. Find $p(X = 0.2/X < 0.6)$
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Exercice 0.4. I roll two dice and observe two numbers X and Y

- Find S_X, S_Y and the PMFs of X and Y
 - Find $p(X = 2, Y = 6)$
 - Find $p(X > 3/Y = 2)$
 - Let $Z = X + Y$ Find the sample space and PMF of Z
 - Find $p(X = 3/Z = 8)$
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Exercice 0.5. We roll a fair die repeatedly until a number larger than 4 is observed. If N is the total number of times that I roll the die, find $P(N=k)$, for $k = 1, 2, 3, \dots$

Exercice 0.6. You take an exam that contains 20 multiple-choice questions. Each question has 4 possible options. You know the answer to 10 questions, but you have no idea about the other 10 questions so you choose answers randomly. Your score X on the exam is the total number of correct answers. Find the PMF of X . What is $p(X > 15)$?

Exercice 0.7. Let $X \sim \text{Poisson}(\alpha)$ and $Y \sim \text{Poisson}(\beta)$ be two independent random variables. Define a new random variable as $Z = X + Y$. Find the PMF of Z
