

Homework

Exercise 0.1.

- The density function representing the shelf life of a product in days is provided as follows:

$$f_X(x) = \begin{cases} k \frac{10000}{(x+50)^3} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

1. Find $E[X]$ and $Var(X)$.
 2. Find the probability that a piece of this product will have a shelf life of:
 - (a) at least 200 days
 - (b) anywhere from 80 to 120 days.
- Use axioms of the algebra of events to prove the relations:
 1. $A \cap A = A$
 2. $A \cap (A \cup B) = A$
 - Consider the Markov chain with three states, $S = \{1, 2, 3\}$, that has the following (incomplete) transition matrix:

$$\begin{pmatrix} ? & 0.2 & 0.5 \\ 0 & ? & 0.5 \\ 0.3 & 0.2 & ? \end{pmatrix}$$

1. Fill in the missing values in the transition matrix.;
 2. Draw the Markov chain.;
 3. Obtain the steady state probabilities.
 4. If we know $P(X_0 = 1) = P(X_0 = 2) = \frac{1}{4}$, find $P(X_1 = 3), P(X_2 = 2), P(X_3 = 1)$
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