

# Asymmetric Cryptography

## Part 2

Pr. Nouredine Chikouche

[nouredine.chikouche@univ-msila.dz](mailto:nouredine.chikouche@univ-msila.dz)

<https://sites.google.com/view/chikouchenouredine>

# Course outline

- Key exchange
- RSA Key exchange protocol
- Diffie-Hellman Protocol
- Comparison

# Key exchange

- ▶ Internet protocols make extensive use of session keys.
- ▶ A **session key** is a symmetric **temporary** key used to protect the transmitted data by the symmetric encryption during a communication session.

# Key exchange

- ▶ To exchange session keys, two methods can be used:
  - ▶ **Secure channel:** conversation, VPN, ...
  - ▶ **Unsecured (public) channel:** Internet, Wi-Fi, mobile phones, ...
- ▶ Public key algorithms (like RSA) are not used for data encryption, but rather for the exchange of **keys session** for symmetric cryptography.

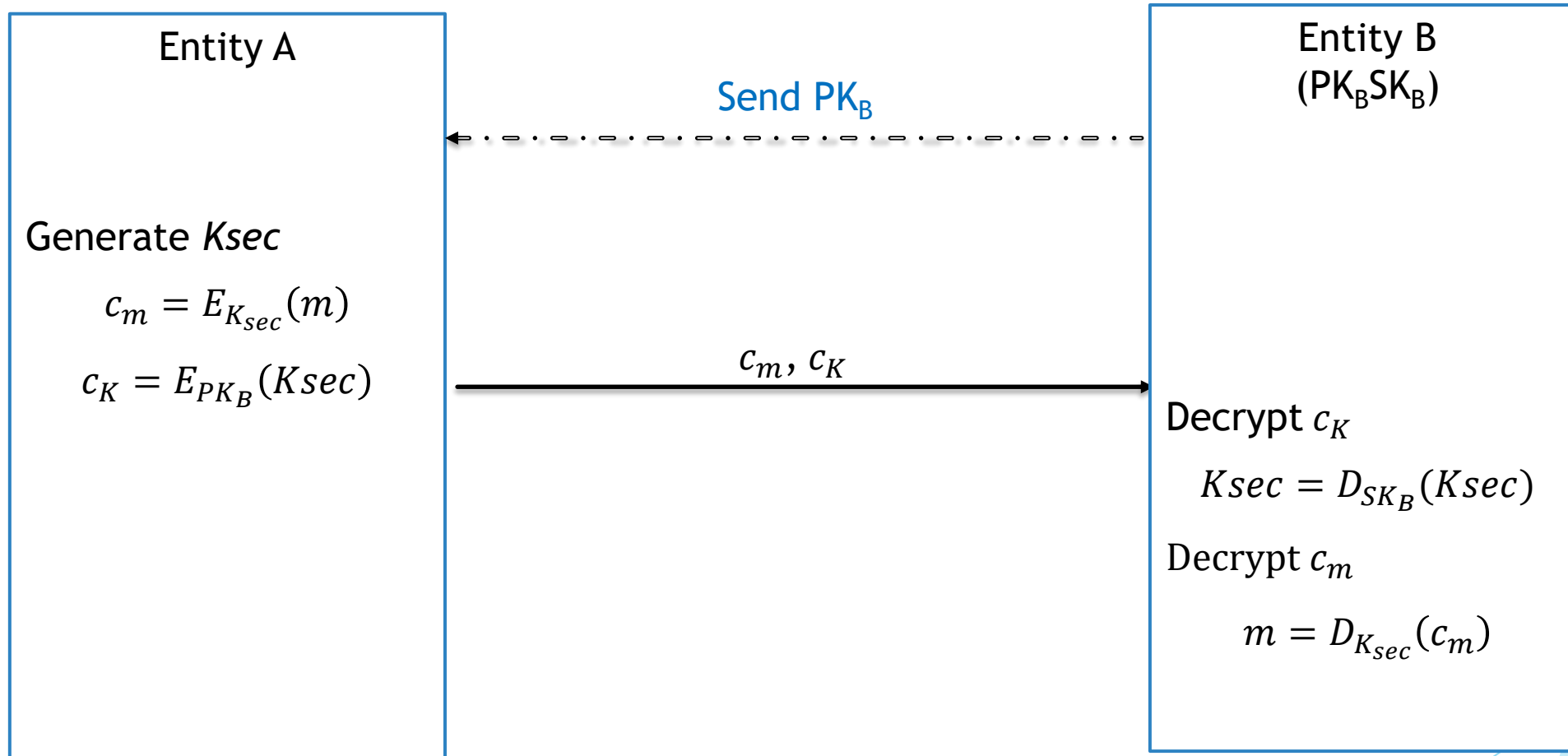
# Key exchange

- ▶ For exchange keys, we use the **key exchange protocols**.
- ▶ There are two categories of key exchange protocols:
  - ▶ Use knowledge previously shared between communicating entities. For example, **RSA key exchange protocol**.
  - ▶ Do not use knowledge previously shared between the communicating entities. For example, **Diffie-Hellman key exchange protocol**.

# RSA key exchange Protocol

- ▶ The information previously exchanged between the entities is **the recipient's public key**.
- ▶ This protocol is used in HTTPS and PGP protocols.
- ▶ We use the principle of **hybrid encryption**:
  - ▶ Encrypt the session key using the recipient's public key.
  - ▶ Encrypt the message to be sent using the session key (symmetric encryption).

# RSA Key Exchange Protocol



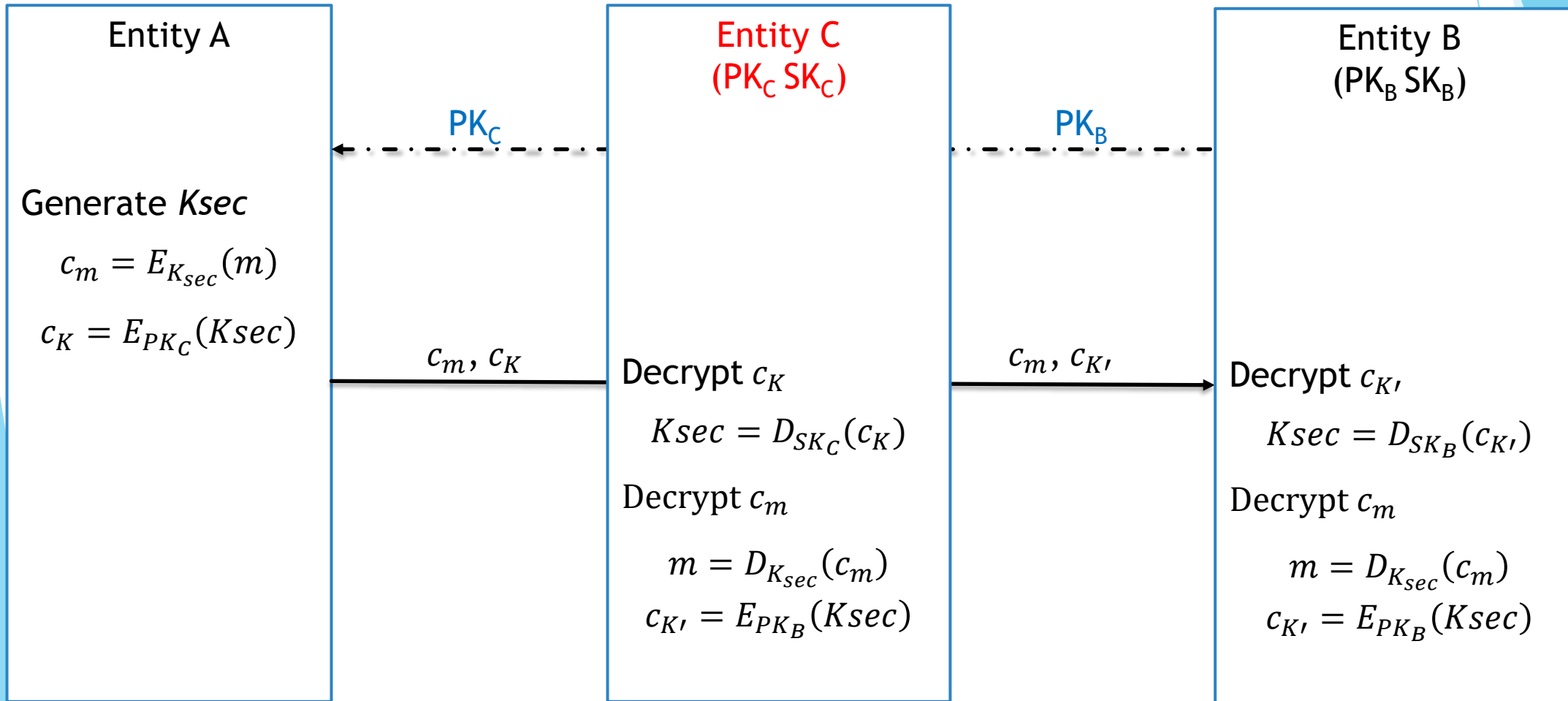
# RSA key exchange Protocol

- ▶ Entity A randomly chooses a session key  $K_{sec}$  (ex. of size 128 bits for AES-128).
- ▶ A encrypts the message to be sent using the key  $K_{sec}$  using the symmetric encryption function,  $c_m = E_{K_{sec}}(m)$
- ▶ A encrypts the session key using the recipient's public key ( $PK_B$ ),  $c_K = E_{PK_B}(K_{sec})$
- ▶ A sends  $c_m$  and  $c_K$  to entity B.
- ▶ B decrypts  $c_K$  using their private key to obtain the session key  $K_{sec} = D_{SK_B}(c_K)$
- ▶ B decrypts the encrypted message using the received session key  $m = D_{K_{sec}}(c_m)$ .



# RSA Key Exchange Protocol

## Attack



# RSA key exchange Protocol

## Attack

- ▶ The RSA key exchange protocol is vulnerable to **active attacks** (*man-in-the-middle*).
- ▶ Principle of the attack:
  - ▶ During the exchange of the public key of  $B$  the attacker  $C$  replaces  $PK_B$  by its public key ( $PK_C$ ) and sends it to entity  $A$ .
  - ▶ The entity  $A$  uses the public key of  $C$  to encrypt the session key. Then, the attacker can simply obtain the session key during the communication between  $A$  and  $B$ .
- ▶ To avoid this vulnerability, it is necessary to **certify the recipient's public key**.

# Diffie-Hellman Protocol



Diffie & Hellman

- ▶ First public key scheme proposed in 1976.

# Diffie-Hellman Protocol

- ▶ This protocol is more commonly used for exchanging a session key without needing the previously shared information.
- ▶ It is often used in commercial products (SSL,...).
- ▶ Not authenticated in the basic version.
- ▶ The security of the DH protocol relies on the problem of **discrete logarithm** and on **the problem of DH**.

# Diffie-Hellman Protocol

## discrete logarithm

- ▶ Either  $G$  a cyclic group. Let  $p$  a prime number and  $g$  a generator of  $\mathbb{Z}_p^*$ .
- ▶ Given  $x$  calculate  $y = g^x \bmod p$ . This is easily done by rapid exponentiation.
- ▶ Problem: Find  $x = \log_g(y) \bmod p-1$ . Knowing  $y$ ,  $g$  and  $p$ .
- ▶ **Applications:** El-Gamal cryptosystem and Diffie-Hellman protocol.

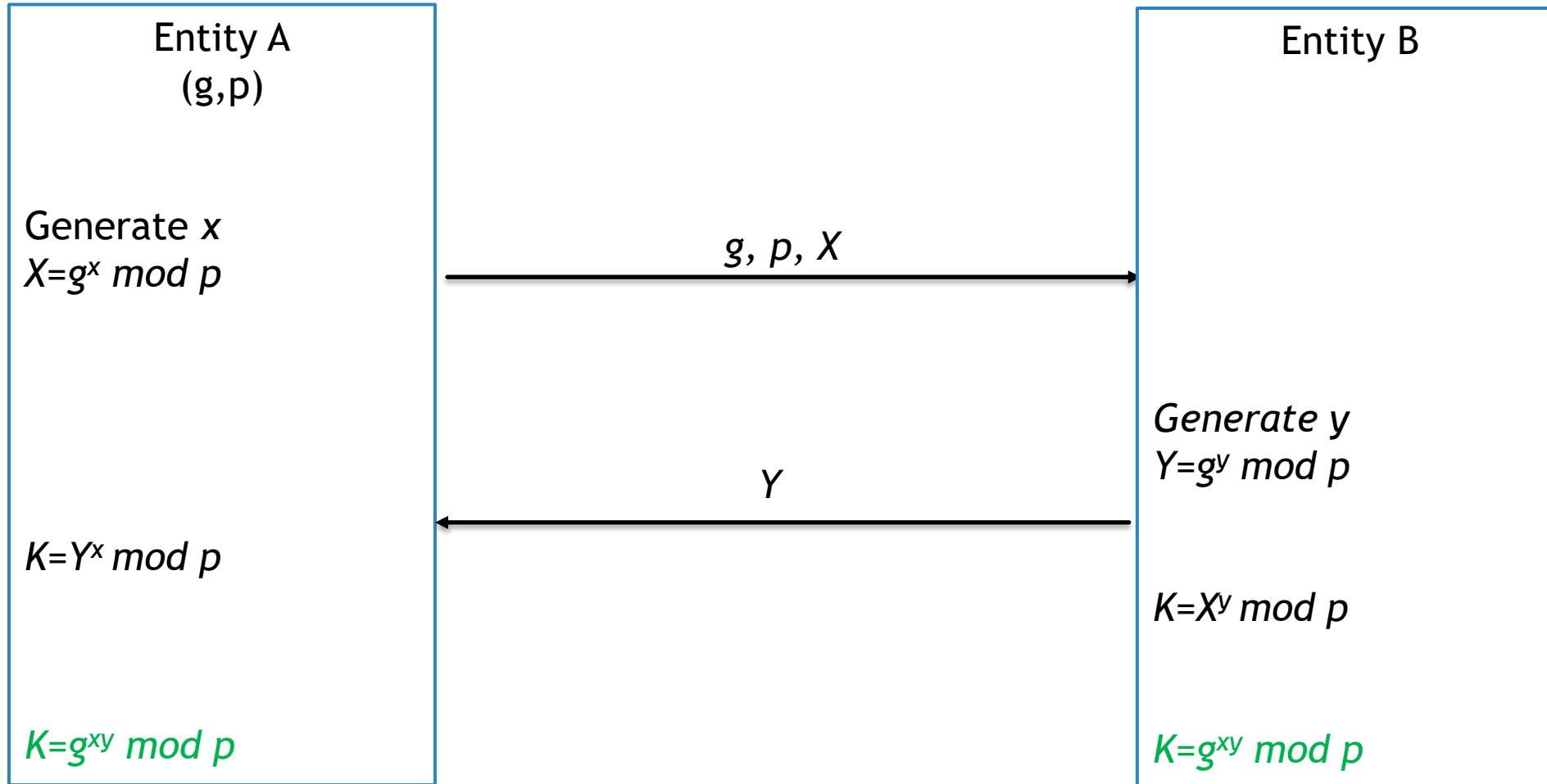
# Diffie-Hellman Protocol

## Basic version

- ▶ The problem of DH means the difficulty of calculating  $k = g^{xy} \bmod p$  from  $X = g^x \bmod p$  and  $Y = g^y \bmod p$ , when  $p$  is big prime number.
- ▶  $x$  and  $y$  are private keys.
- ▶  $X$  and  $Y$  are public keys.

# Diffie-Hellman Protocol

## Basic version



# Diffie-Hellman Protocol

## Basic version

- ▶ Either  $p$  a big prime number and  $g$  is a generator of  $Z_p^*$ .
- ▶ A selects an integer  $x$  and calculate  $X = g^x \bmod p$ .
- ▶ A Sends  $X$ ,  $g$ , and  $p$  to B.
- ▶ B selects an integer  $y$  and calculate  $Y = g^y \bmod p$ .
- ▶ B sends  $Y$  to A.



# Diffie-Hellman Protocol

## Basic version

- ▶ Entity A calculates  $(g^y \bmod p)^x \bmod p = g^{xy} \bmod p$
- ▶ Entity B calculates  $(g^x \bmod p)^y \bmod p = g^{xy} \bmod p$
- ▶ The shared secret key is:

$$K = g^{xy} \bmod p$$

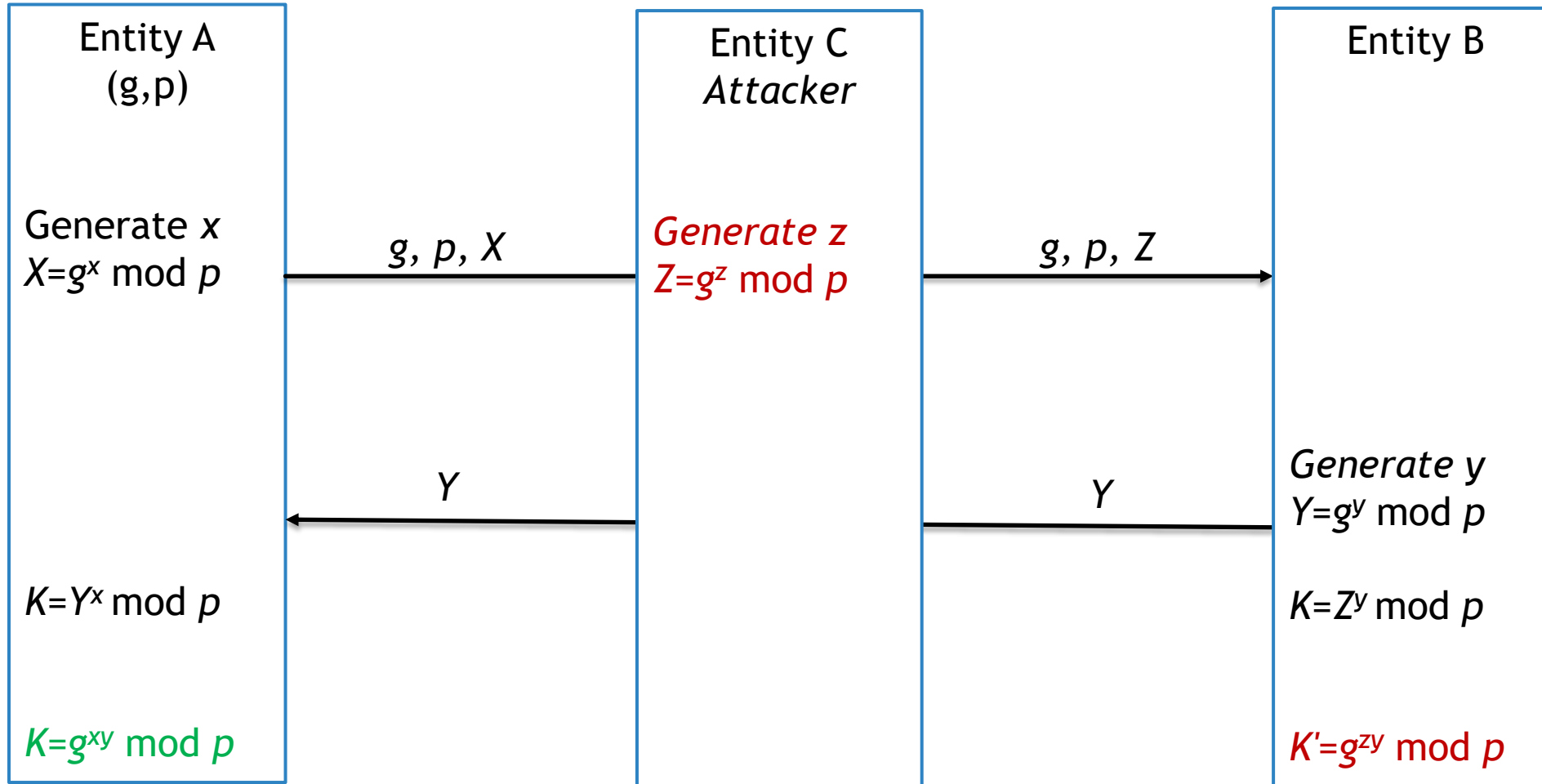
# Diffie-Hellman Protocol

## Basic version

- ▶ Example:
- ▶ We assume that entities A and B share  $p = 233$  and  $g = 45$ :
- ▶ A selects  $x = 11$  and Bob  $y = 20$ , so:
  - ▶  $X = 45^{11} \bmod 233 = 147$ ,
  - ▶  $Y = 45^{20} \bmod 233 = 195$ ,
- ▶  $Y^x \bmod p = 195^{11} \bmod 233 = 169$
- ▶  $X^y \bmod p = 147^{20} \bmod 233 = 169$ .
- ▶ Entities A and B share a secret key,  $k = 169$ .

# Diffie-Hellman Protocol

## Attack 1



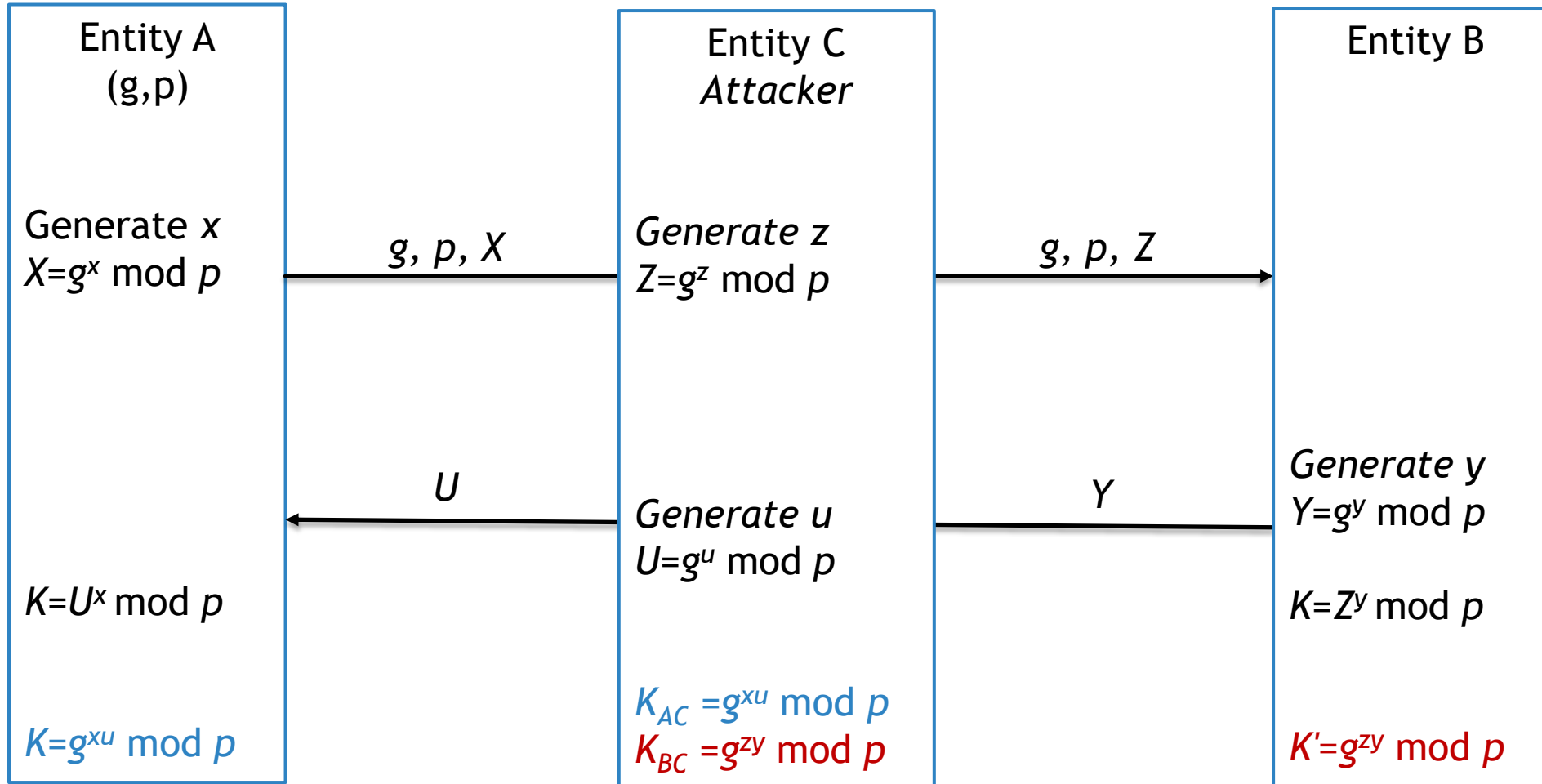
# Diffie-Hellman Protocol

## Attack 1

- ▶ The DH protocol is vulnerable to **active attacks** (*man-in-the-middle*).
- ▶ During the exchange of keys, the attacker *C* blocks message *X* sent by *A*, selects a random number *z* and calculate  $Z = g^z \bmod p$ . *C* sends message *Z* to entity *B*.
- ▶ **Result:** the two entities *A* and *B* calculate **two different keys**,  $k_{HAS} = g^{xy} \bmod p$  for entity *A* and  $k_B = g^{yz} \bmod p$  for entity *B*.
- ▶ When entity *A* sends an encrypted message *m* using its secret key, entity *B* cannot obtain the clear message from *A*.

# Diffie-Hellman Protocol

## Attack 2



# Diffie-Hellman Protocol

## Attack 2

- ▶ The DH protocol is vulnerable to **attacks active** (*man-in-the-middle*):
- ▶ During the exchange of keys, the attacker *C* modifies sent messages *X* and *Y* and replaces them with new messages *Z* and *U*, respectively.
- ▶ Entities A and B create different session keys, *K* and *K'*.
- ▶ The attacker creates keys  $K_{AC}$  and  $K_{BC}$  such as:  $K_{AC} = K$  and  $K_{BC} = K'$ .

# Diffie-Hellman Protocol

## Attack 2

- ▶ When  $A$  sends an encrypted message  $E_K(m)$  to  $B$ , the attacker intercepts the message sent, then decrypts it using the same key  $K_{AC}$ .
- ▶ However, Entity  $B$  cannot obtain the clear message because of  $K$  and  $K'$  are different.
- ▶ The attacker  $C$  can apply the same scenario with  $B$ .

# Diffie-Hellman Protocol

## Advantages & Disadvantages

### ▶ Benefits:

- ▶ The discrete logarithm problem is difficult.

### ▶ Disadvantages:

- ▶ An attacker can observe:  $p$ ,  $g$ ,  $g^b \bmod p$  and  $g^a \bmod p$
- ▶ Like RSA, very slow.
- ▶ Absence of authentication in the basic version



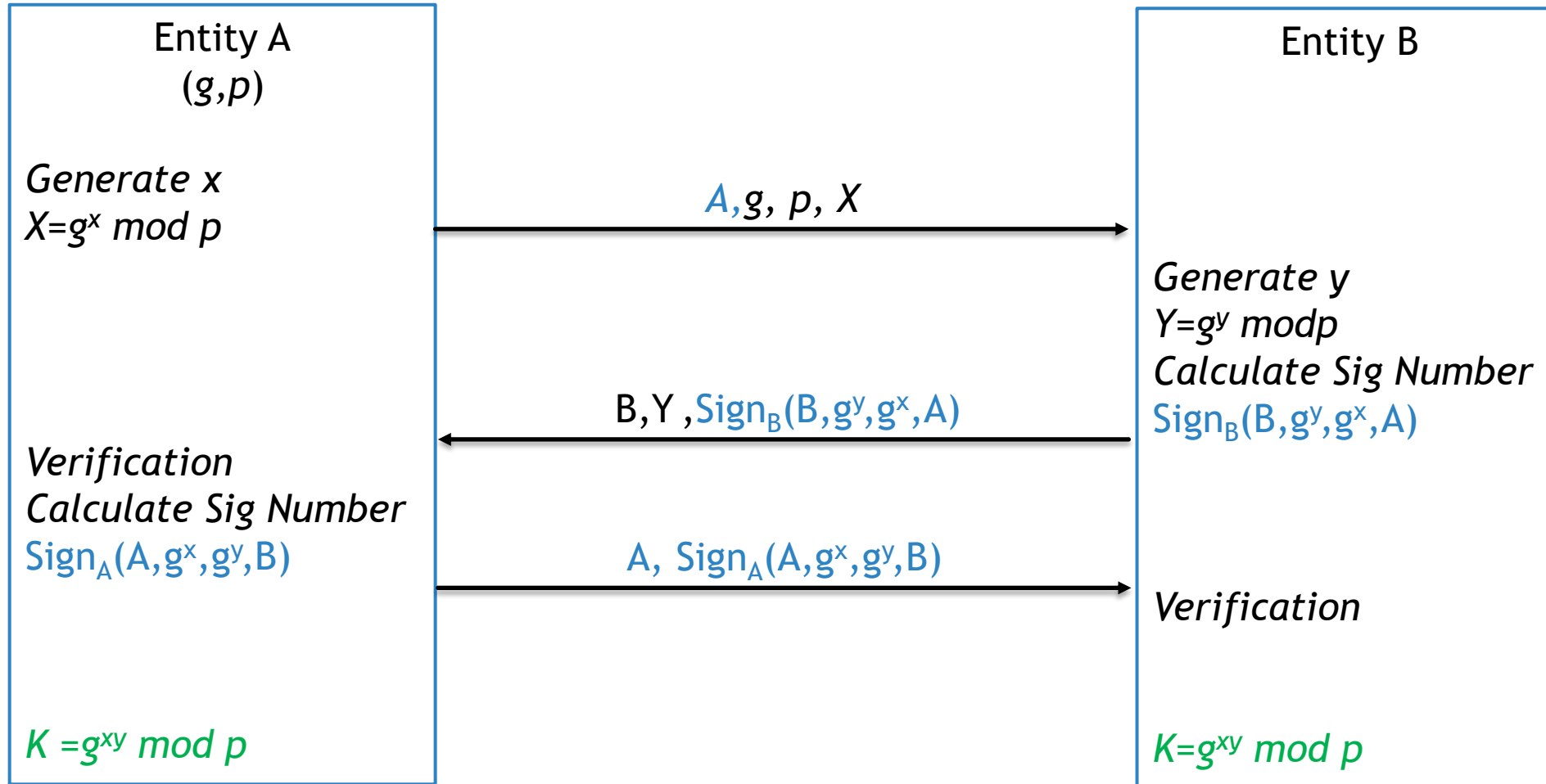
# Diffie-Hellman Protocol

## Signed version

- ▶ To avoid the attack through the middle, the solution is to use digital signature.
- ▶ It consists of the fact that:
  - ▶ Entity A sends  $(A, g^x)$ ,
  - ▶ B sends  $(B, g^y, \text{Sign}_B(B, g^y, g^x, A))$
  - ▶ A verifies the received signature and calculate the secret key  $K$ .
  - ▶ Finally, A calculates his signature  $(A, \text{Sign}_A(A, g^x, g^y, B))$  and sends it to B.
  - ▶ B checks the signature received and calculates the secret key  $K$ .

# Diffie-Hellman Protocol

## Signed version



# Comparison

## Symmetric encryption vs Asymmetric encryption

|               | Symmetric                                                                                                                                                                                                                        | Asymmetric                                                                                                                                                                                                       |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Benefits      | <ul style="list-style-type: none"><li>• It is faster in execution and requires less computing power.</li><li>• It uses small keys (64 - 256) bits.</li><li>• It is easily implemented on the hardware.</li></ul>                 | <ul style="list-style-type: none"><li>• Allow messages to be signed.</li><li>• Facilitate key distribution.</li><li>• It requires <math>2n</math> keys only for <math>n</math> entities communicators.</li></ul> |
| Disadvantages | <ul style="list-style-type: none"><li>• It does not provide certain services (e.g. signature).</li><li>• Key distribution.</li><li>• It requires <math>n(n-1)/2</math> keys for <math>n</math> entities communicators.</li></ul> | <ul style="list-style-type: none"><li>• He is slow to execute (1000 times slower).</li><li>• Key size greatly increased.</li></ul>                                                                               |