Chapter 3: introduction aux méthodes multicritères

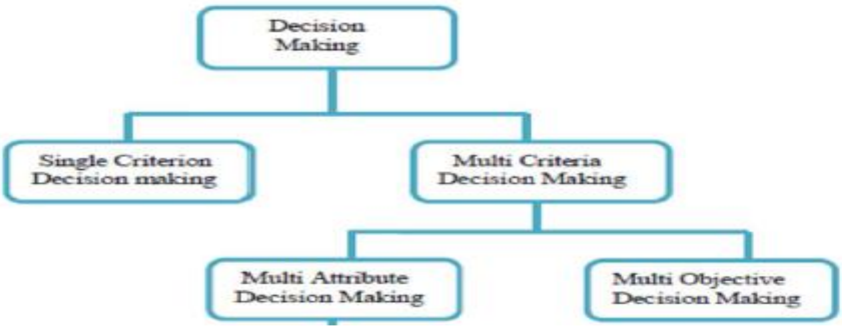
1. Multiple-criteria decision-making or multiple-criteria decision analysis (MCDA)

Multiple criteria decision-making (MCDM) refers to making decisions in the presence of multiple - usually conflicting criteria. The aim of studying this text is not to be familiar with each existing method (impossible due to high number of methods and continual development of new), but understanding of basic principles and ability to use some methods (via software) and explain results.

2. Classification of Decision Making Models

MCDM problems are generally categorized as continuous or discrete, depending on the domain of alternatives. Hwang and Yoon (1981) have classified the MCDM methods into two categories: multi-objective decision-making (MODM) and multi-attribute decision-making (MADM). MODM is commonly used to solve problems characterized by multiple and conflicting objective functions such as maximizing performance while minimizing fuel consumption of a vehicle simultaneously over a feasible set of decisions. An MODM model considers a vector of decision variables, objective functions, and constraints and the goal is to optimize the objective functions. The Decision Maker chooses a solution among a set of efficient solutions since MODM problems rarely have a unique solution. Goal Programming, Simulated Annealing, Evolutionary Algorithms such as Genetic Algorithm and Hybrid approaches are examples of MODM

MADM methods, on the other hand, have been used to solve problems with discrete decision spaces and a predetermined or a limited number of alternative choices.



3. Example of MODM Linear programming problem

A company makes two products (X and Y) using two machines (A and B). Each unit of X that is produced requires 50 minutes processing time on machine A and 30 minutes processing time on machine B. Each unit of Y that is produced requires 24 minutes processing time on machine A and 33 minutes processing time on machine B.

At the start of the current week there are 30 units of X and 90 units of Y in stock. Available processing time on machine A is forecast to be 40 hours and on machine B is forecast to be 35 hours.

The demand for X in the current week is forecast to be 75 units and for Y is forecast to be 95 units. Company policy is to maximise the combined sum of the units of X and the units of Y in stock at the end of the week.

Solution

Let
x be the number of units of X produced in the current week
y be the number of units of Y produced in the current week
then the constraints are:
$50x + 24y <= 40(60)$ machine A time
$30x + 33y <= 35(60)$ machine B time
$x >= 75 - 30$
i.e. $x >= 45$ so production of X >= demand (75) - initial stock (30), which ensures we meet demand
$y >= 95 - 90$
i.e. $y >= 5$ so production of Y >= demand (95) - initial stock (90), which ensures we meet demand
The objective is: maximise $(x+30-75) + (y+90-95) = (x+y-50)$
i.e. to maximise the number of units left in stock at the end of the week

4. Basic Concepts

All MCDM problems have the following common characteristics:

- Multiple objectives/attributes. Some attributes may break down further into lower levels of attributes, called sub-attributes. Attributes often form a hierarchy.
- Each problem has multiple objectives/ attributes. A decision-maker must generate relevant objectives/attributes for each problem setting.
- Conflict among criteria
- Multiple criteria usually conflict with each other (for example the quality and price of goods).
- Incommensurable units. Each objective/attribute has a different unit of measurement. In the car selection case, gas mileage is expressed by miles per gallon (MFG), comfort is by cu ft if fit is measured by passenger space, safety may be indicated in a nonnumeric way, cost is indicated by dollars, etc.
- Mixture of qualitative and quantitative attributes. It is possible that some attributes can be measured numerically and other attributes can only be described subjectively. For instance, the price of a car is numerical and the comfort rating is qualitative.
- Mixture of deterministic and probabilistic attributes. For example, in the car selection problem, car price is deterministic and fuel economy could be random. Fuel economy changes depending on road conditions, traffic conditions and weather.
- Design/selection: Solutions to these problems are either to design the best alternative or to select the best one among previously specified finite alternatives.
- Uncertainty
  - Uncertainty in subjective judgments. It is common that people may not be 100% sure when making subjective judgments.
  - Uncertainty due to lack of data or incomplete information. Sometimes information of some attributes may not be fully available or even not available at all.

5. Decision matrix

The MADM problem can be expressed in a matrix containing criteria in columns and variants in lines. Performance Matrix

$$
\begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{array}
\begin{array}{c}
\begin{array}{cccc} f_1 & f_2 & \cdots & f_m \end{array} \\
\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix}
\end{array}
$$

Decision (Criteria) Matrix for MADM Model

6. Decision-Making Process

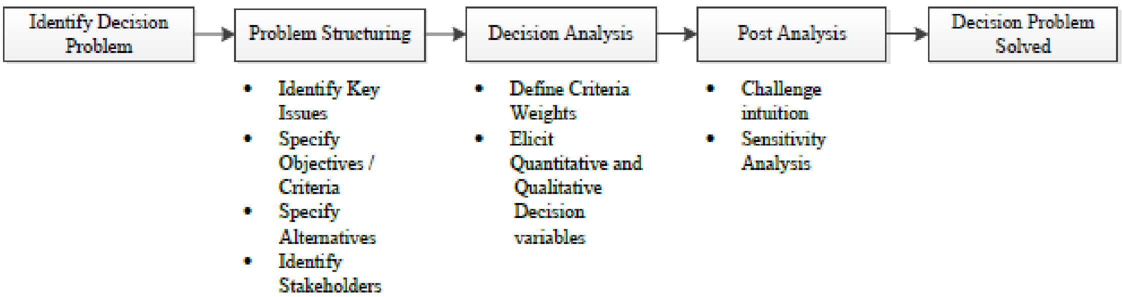


Figure 2-6 Decision-Making Process, adapted from Belton & Stewart (2010); Franco & Montibeller (2009)

7. MCDM Solutions

MCDM problems may not always have a conclusive or unique solution. Therefore different names are given to different solutions depending on the nature of the solutions [Hwang and Yoon, 1981].

7.1. Ideal solution

All criteria in a MCDM problem can be classified into two categories. Criteria that are to be maximized are in the profit criteria category, although they may not necessarily be profit criteria. Similarly criteria that are to be minimized are in the cost criteria category.
An ideal solution to a MCDM problem would maximize all profit criteria and minimize all cost criteria. Normally this solution is not obtainable. The question is what would be a best solution for the decision maker and how to obtain such a solution?

7.2. Non dominated solutions

If an ideal solution is not obtainable, the decision maker may look for non-dominated solutions. An alternative (solution) is dominated if there are other alternatives that are better than the solution on at least one attribute and as good as it on other attributes. An alternative is called non-dominated if it is not dominated by any other alternatives.

7.3. Satisfying solutions

A satisfying solution is a reduced subset of the feasible set which exceeds all of the aspiration levels of each attribute. A set of satisfying solutions is composed of acceptable alternatives. A satisfying solution may well be used as the final solution though it is often utilized for screening out unacceptable solutions. A satisfying solution may not be a nondominated solution. Whether a solution is satisfying depends on the level of the decision maker's expectation.

7.4. Preferred solutions

A preferred solution is a non-dominated solution that best satisfies the decision maker's expectations.

8. MCDM Methods

There are two types of MCDM methods. One is compensatory and the other is noncompensatory [Hwang and Yoon, 1981].

Table 2.1: Typical Non-compensatory and Compensatory Decision Analysis Methods

Non-compensatory Methods	Compensatory Methods
Conjunctive method	Analytic hierarchy process
Disjunctive method	Expected utility theory
Dominance method	Multi-attribute utility theory
ELECTRE	Multiplicative weighting method
Elimination by aspects	PROMETHEE
Lexicographic method	Simple additive weighting
Maximin method	TOPSIS
Maximax method	

8.1 Non-compensatory Methods

These models do not permit trade-offs between attributes. An advantage or favourable value in some other attribute cannot offset a disadvantage or unfavourable value in one attribute.

Hence comparisons are made on an attribute-by-attribute basis. The MADM methods that belong to this model are credited for their simplicity. The MCDM methods in this category are credited for their simplicity. Examples of these methods include: dominance, maximin, maximax, conjunctive constraint method, disjunctive constraint method, and lexicographic method.

8.1.1 Dominance method: Eliminate all dominated alternatives. There could be more than one solutions generated by this method.

There are no assumptions so we don't need to transfer minimizing criteria to maximizing. Steps of the method:

- Compare first two alternatives and if one is dominated by other, discard the dominated one.
- Compare the undiscarded alternatives with third alternative and discard any dominated alternative

The nondominated set is determined after (m-1) comparisons

8.1.2 Maxmin method: Find the weakest attribute value (min) of each alternative and then choose the alternative with the best (max) weakest attribute value. The logic is that a chain is as strong as its weakest link. This method is applicable only when attribute values are comparable with one another, either measured in the same unit or transformed to a common scale.

8.1.3 Maxmax Method: In contrast to the Maxmin method, the Maxmax method selects an alternative by its best attribute value. It is also applicable only when attributes are comparable.

8.1.4 Conjunctive constraint method: By setting up a minimum standard for each attribute, the alternative selection or evaluation process is simplified to compare each attribute against its standard. If the standard reflects the decision maker's expectations, the obtained solutions are satisfying solutions.

The conjunctive (satisfying) method is not usually used for selection of alternatives but rather for dividing them to acceptable and not acceptable.

8.1.5 Disjunctive constraint method: This method evaluates an alternative on its best attribute regardless of all other attributes.

8.1.6 Lexicographic Method: In some decision situations a single attribute seems to predominate. One way of treating this situation is to compare the alternatives on the most important attribute. If one alternative has a higher attribute value than any of the other alternatives, the alternative is chosen and the decision process ends.

However, if some alternatives are tied on the most important attribute, the subsets of tied alternatives are then compared on the next most important attribute. The process continues sequentially until a single alternative is chosen or until all n attributes have been considered.

8.2 Compensatory Model

Compensatory models permit trade-offs between attributes. That is, in these models changes (perhaps small changes only) in one attribute can be offset by opposing changes in any other attributes. With compensatory models a single number is usually assigned to each multidimensional characterization representing an alternative. Based upon the principle of calculating this number, compensatory models can be divided into 4 subgroups.

8.2.1 Scoring Methods

The scoring method selects or evaluates an alternative according to its score (or utility). Utility or score is used to express the decision maker's preference. It transforms attribute values into a common preference scale such as [0,1] so that comparisons between different attributes becomes possible.

A very popular method in this category is the Simple Additive Weighting method. This method calculates the overall score of an alternative as the weighted sum of the attribute scores or utilities.

The Analytical Hierarchy Process (AHP) is another popular method in this category.
This method calculates the scores for each alternative based on pairwise comparisons/

8.2.2 Compromising Methods

The compromising method selects an alternative that is closest to the ideal solution. The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method belongs to this category. This method first normalizes the decision matrix of a MCDM problem. Then based on the normalized decision matrix, it calculates the weighted distances of each alternative from an ideal solution and a nadir (worst) solution. A solution relatively close to the ideal solution and far from the nadir solution is evaluated to be the best [Hwang and Yoon, 1981].

8.2.3 Concordance Methods

The concordance method generates a preference ranking which best satisfies a given concordance measure. The Linear Assignment Method is one of the examples in this family. In this method it is believed that an alternative having many highly ranked attributes should be ranked high [Hwang and Yoon, 1981].

9. Normalization

Units of measurement of quantitative multiple criteria can be different: per cent, ranks, grades, money units, physical units, etc. Consequently, their incorporation into a single evaluation criterion is possible if values of criteria are independent of units of measurement. Such dimensionless values are obtained by normalizing the values.
Criteria can be both minimizing and maximizing. Some MCDA methods imply transformation of minimizing criteria into maximizing ones.

Some MCDA methods can use only positive criteria. Therefore, majority of MCDA methods use both normalization and transformation of criteria with negative values.

In general, data normalization and data standardization means mapping the data values to a common scale, usually within the unity interval [0, 1]. Normalization is a transformation process to obtain numerical and comparable input data by using a common scale.

Among the well known normalization techniques are: Vector Normalization, Linear Max-Min Normalization, Linear Sum based Normalization, Linear Max Normalization and Gaussian Normalization

a) Vector Normalization

The original decision matrix can be normalized using the vector normalization method.

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^2}} \quad \text{for} \quad i = 1, \ldots, m; j = 1..n$$

b) Linear Max-Min Normalization

The Linear Max-Min Normalization technique has the following general form:

The normalized value r_{ij} for benefit criteria is obtained by

$$r_{ij} = \frac{a_{ij} - a_j^{min}}{a_j^{max} - a_j^{min}} \quad \text{for} \quad i = 1, \ldots, m$$

$$j = 1, \ldots, n$$

The normalized value r_{ij} for cost criteria is computed as

$$r_{ij} = \frac{a_j^{max} - a_{ij}}{a_j^{max} - a_j^{min}} \quad \text{for} \quad i = 1, \ldots, m;$$

$$j = 1, \ldots, n$$

c) Linear Sum Based Normalization

Normalize the decision matrix by using the Linear Sum based Normalization technique. This method divides the rating of each alternative by the sum of rating of each criterion as follows

$$r_{ij} = \frac{a_{ij}}{\sum_{j=1}^{n} a_j} \qquad \text{for} \quad i = 1, \dots, m;$$

$$j = 1, \dots, n$$

d) Linear Max Normalization

The Linear Max Normalization technique has the following general form:

The normalized value r_{ij} for benefit criteria is obtained by

$$r_{ij} = \frac{a_{ij}}{a_j^{max}} \qquad \text{for} \quad i = 1, \dots, m;$$

$$j = 1, \dots, n$$

The normalized value r_{ij} for cost criteria is computed as

$$r_{ij} = 1 - \frac{a_{ij}}{a_j^{max}} \qquad \text{for} \quad i = 1, \dots, m;$$

$$j = 1, \dots, n$$

e) Gaussian Normalization

Normalize the decision matrix by using the Gaussian Normalization technique. Two factors are taken into consideration that leads to the rating variance of the users with similar interests.

a) Average rating shift

b) Different rating scales

The above two ideas are combined together, to normalize the rating of each alternativei based on the criteria j is computed by:

$$r_{ij} = \frac{a_{ij} - \bar{a}_i}{\sqrt{\sum_{j=1}^{n} (a_{ij} - \bar{a}_i)^2}}$$

$$\text{for} \quad i = 1, \dots, m; \quad j = 1, \dots, n$$

Where a_{ij} - represent the original rating of each alternative based on the criteria. $\bar{a}_i$- stands for the average rating of alternative i.

Table 1 Normalisation techniques

Normalisation technique	*Condition of use*	*Formula*
Linear: Max (N_1) (Celen, 2014)	Benefit criteria	$n_{ij} = \frac{r_{ij}}{r_{\max}}$
	Cost criteria	$n_{ij} = 1 - \frac{r_{ij}}{r_{\max}}$
Linear: Max-Min (N_2) (Patro and Sahu, 2015)	Benefit criteria	$n_{ij} = \frac{r_{ij} - r_{\min}}{r_{\max} - r_{\min}}$
	Cost criteria	$n_{ij} = \frac{r_{\max} - r_{ij}}{r_{\max} - r_{\min}}$
Linear: sum (N_3) (Jahan and Edwards, 2014)	Benefit criteria	$n_{ij} = \frac{r_{ij}}{\sum_{i=1}^{m} r_{ij}}$
	Cost criteria	$n_{ij} = \frac{1/r_{ij}}{\sum_{i=1}^{m} 1/r_{ij}}$
Vector normalisation (N_4) (Jahan and Edwards, 2014)	Benefit criteria	$n_{ij} = \frac{r_{ij}}{\sqrt{\sum_{i=1}^{m} r_{ij}^2}}$
	Cost criteria	$n_{ij} = 1 - \frac{r_{ij}}{\sqrt{\sum_{i=1}^{m} r_{ij}^2}}$
Logarithmic normalisation (N_5) (Jahan and Edwards, 2014)	Benefit criteria	$n_{ij} = \frac{\ln(r_{ij})}{\ln\left(\prod_{i=1}^{m} r_{ij}\right)}$
	Cost criteria	$n_{ij} = \frac{1 - \frac{\ln(r_{ij})}{\ln\left(\prod_{i=1}^{m} r_{ij}\right)}}{m-1}$
Fuzzification (N_6) (Ribeiro, 1996)	Benefit and cost criteria	Using membership function (e.g., trapezoid)

Notes: *r_{ij} is the rating of alternative *i* with respect to criterion *j*.
**n_{ij} is the normalised value of r_{ij}.

Example Colour Films

Original decision matrix :

Variant	f1(price)	f2	f3	f4	f5
1.	170,00	28,80	3,00	1,00	94,00
2.	165,00	30,20	4,00	1,50	105,00
3.	173,00	28,30	3,00	1,00	119,00
4.	168,00	27,60	4,00	0,50	105,00

Transform to all maximal criteria:

Variant	f1	f2	f3	f4	f5
1.	3	28,80	3,00	1,00	94,00
2.	8	30,20	4,00	1,50	105,00
3.	0	28,30	3,00	1,00	119,00
4.	5	27,60	4,00	0,50	105,00

Transformation to All Maximal Criteria –

This transforms no beneficial attribute to beneficial attribute. A benefit criteria, that is, the higher the values are, the better it is.

STEP 1: In each column find the column maximum (marked in the above table):

$$H_j = \max_i d_{ij} \quad i=1,2,\ldots,m$$

STEP 2: Transforrm each element in each column. The transformed decision matrix A thereinafter:

$$a_{ij} = H_j - d_{ij} \quad i=1,2,\ldots,m \quad j=1,2,\ldots,n$$

Discarding Dominated Variants We have found that A1 and A4 are dominated variants, so that we can overpass them. As the maximin method requires common scale for all criteria we transform the decision matrix according:

$$r_{ij} = \frac{a_{ij}}{a_j^*}, \textit{ where } a_j^* = \max a_{ij} \textit{ for } i = 1, \ldots, n$$

Variant	f1	f2	f3	f4	f5
2.	1	1	1	1	0,882
3.	0	0,937	0,75	0,667	1

Maximin

The line minima are: A2: 0.882 and A4: 0

Maximum of these minima is 0.882, so we chose the variant A2.

MaxiMax

We use the normalized decision matrix with all maximal criteria:
The line maxima are: A2: 1 and A4: 1

The line maxima are equal, so according to maximax method we are not able to distinguish which of A2 and A4 variants are better.

10. Weights Assessing

Many methods require information about the relative importance of each attribute. It is usually given by a set of (preference) weights, which is normalized to sum to 1.

The majority of authors suggest the classification of the models of determining the weights of criteria into subjective and objective models.

Subjective approaches reflect the decision-maker's subjective opinion and intuition. By such an approach, the decision-maker directly influences the outcome of the decision-making process, since the weights of criteria are determined based on the information obtained from the decision-maker or from the experts involved in the decision-making process.

In a subjective approach, the decision-maker or experts give their opinion on the significance of criteria for a certain decision-making process in accordance with their system of references.

Subjective approaches are mainly based on the pairwise comparisons of criteria or the ranking of criteria.

One of the most commonly applied methods based on pairwise comparisons is the Analytic Hierarchy Process (AHP) method . The methods of pairwise comparisons, in addition to the AHP method, also include the DEMATEL method and the Best Worst Method (BWM) ;

Objective approaches are based on determining the weights of criteria on the basis of the information contained in a decision-making matrix applying certain mathematical models. Objective approaches neglect the decision-maker's opinion.

The most well-known objective methods are the following: the entropy method, the CRITIC method (Criteria Importance Through Intercriteria Correlation) and the FANMA method, named after its authors.