

Exercise 1:

Translate the following English sentences into propositional logic. Define your atomic propositions clearly.

1. If it is raining, then the streets are wet.
2. I will go to the party only if you do.
3. You cannot access the network if you are not a registered user, unless the system is in maintenance mode.
4. The program will terminate if and only if the input is valid and the stack is empty.
5. It is not the case that the light is on and the door is closed.

Exercise 2:

1. Use a truth table to determine if $(P \rightarrow (Q \rightarrow R))$ is logically equivalent to $((P \wedge Q) \rightarrow R)$.
2. Show that $(P \rightarrow Q)$ is logically equivalent to $(\neg P \vee Q)$ using a truth table.
3. Prove using truth tables that “Modus Ponens”, $((P \rightarrow Q) \wedge P) \rightarrow Q$, is a tautology (always true).

Exercise 3:

Using the rules of inference (Modus Ponens, Modus Tollens, Resolution, etc.), prove the validity of the following arguments.

A. Premises: 1. $P \rightarrow Q$ 2. $R \rightarrow \neg Q$ 3. $P \wedge R$ Conclusion: (F) (a contradiction)	B. Premises: 1. $\neg P \rightarrow Q$ 2. $Q \rightarrow R$ 3. $\neg R$ Conclusion: (P)
C. Premises: 1. $A \rightarrow (B \rightarrow C)$ 2. $A \wedge B$ Conclusion:(C)	D. Premises: 1. $S \rightarrow \neg T$ 2. $U \rightarrow T$ 3. $S \wedge U$ Conclusion: F (A contradiction)

Exercises 4:

1. Convert the formula $(P \rightarrow Q) \rightarrow P$ into Conjunctive Normal Form (CNF).
2. Convert the formula $\neg (P \rightarrow Q)$ into CNF.
3. Using the Resolution method, prove that the following set of clauses is unsatisfiable:

$$\{P, Q\} \{P, \neg Q\} \{\neg P, Q\} \{\neg P, \neg Q\}$$

Exercises 5:

Use the DPLL algorithm to check the satisfiability of the following formula in CNF:

1. $((A \vee \neg B) \wedge (\neg A \vee C \vee D) \wedge (\neg A \vee C \vee \neg D) \wedge (\neg C \vee D) \wedge (\neg A \vee \neg C \vee D))$
2. Show your steps: perform unit propagation, make decisions, and backtrack if necessary.