Exn°4

Here we can use The type of proofs

1- proof by induction: (proof by recurrence)

- for $n=1$, $|u^1| = 1|u|$ verified
- " $n=0$ $|u^0| = |\varepsilon| = 0|u|$ verified

Suppose The formula is verified for n

$\Rightarrow |u^n| = n|u| \overset{?}{\Longrightarrow} |u^{n+1}| \overset{?}{=} (n+1)|u|$

$|u^{n+1}| = |u^n . u| = |u^n| + |u| = n|u| + |u| = (n+1)|u|$ verified.

2- proof by Construction:

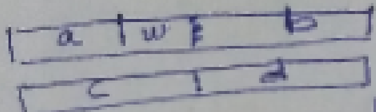
$|u^n| = |u| + |u^{n-1}| = |u| + |u| + |u^{n-2}| = \underbrace{|u| + \dots + |u|}_{n \text{ times}}$

$= n|u|$ (verified).

Exn°5

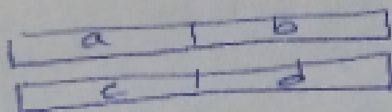
We Can proof This Lemma geometrically:

Case 1: $|a| < |c|$



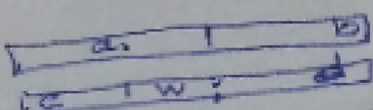
$\exists w \in A^+ : c = aw$ and $b = wd$

Case 2: $|a| = |c|$



$a = c$ and $b = d$

Case 3: $|a| > |c|$



$a = cw$ and $d = wb$