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Faculty of Mathematics and Computer Science

Department: Computer Science

Module:Graph Theory/L2-LMD

TUTORIAL - SERIES NO. 05

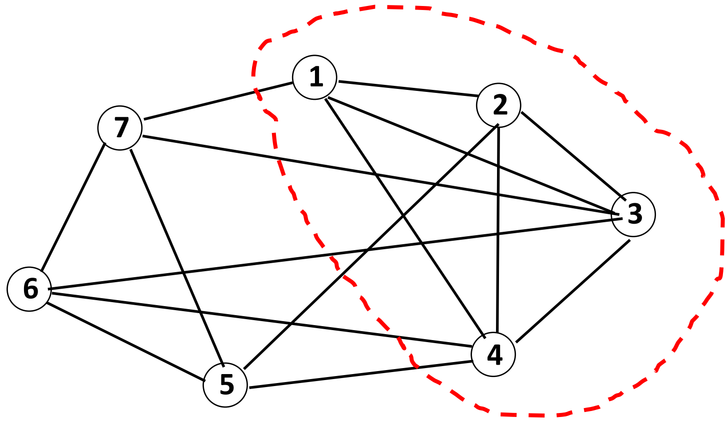
Exercise 01 (Timetable problem)

The courses are numbered from 1 to 7

The following courses have common students: (1, 2), (1, 3), (1, 4), (1, 7), (2, 3), (2, 4), (2, 5), (2, 7), (3, 4), (3, 6), (3, 7), (4, 5), (4, 6), (5, 6), (5, 7), (6, 7).

The graph below represents courses with common students.

How to organize exams in a way that no student has passed two exams for two different courses at the same time



==> course tests having joint students must not organize at the same time.

==> a vertex coloring problem

k-coloring of G with k =γ(G)

{1, 2, 3, 4} is a complete subgraph of G,

==> a maximum clique ==>γ(G)≥4

The stable subsets of G:

S1 = {1, 6}

S2 = {2}

S3 = {3, 5}

S4 = {4, 7}

==>γ(G)≤4 ==>γ(G) = 4

The exams can be divided into 04 periods as follows:

1st period, course tests 1, 6

2nd period, course 2 tests

3rd period, course tests 3, 5

4th period, course tests 4, 7

Solution 2:

Summit	Degree	Color
2	5	C1
3	5	C2
4	5	C3
7	5	C3
1	4	C4
5	4	C2
6	4	C1

The coloring groups are: {2, 6}, {3, 5}, {4, 7}, {1}

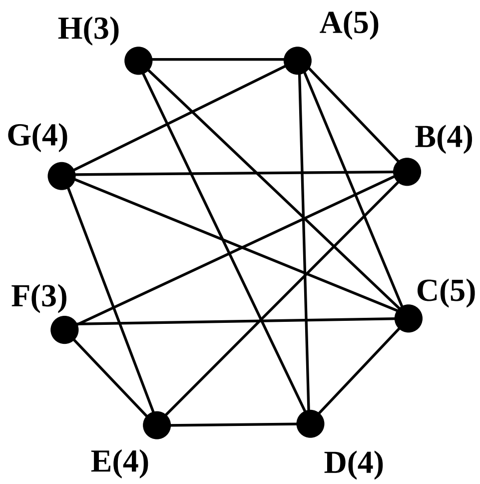
Exercise 02 (An aquarium problem)

A, B, C, D, E, F, G and H denote eight fish; in the table below, a cross means that the fish cannot coexist in the same aquarium:

	A	B	C	D	E	F	G	H
A		X	X	X			X	H
B	X				X	X	X	
C	X			X		X	X	X
D	X		X		X			X
E		X		X		X	X	
F		X	X		X			
G	X	X	X		X			
H	X		X	X				

What is the minimum number of aquariums required?

We draw the incompatibility graph:



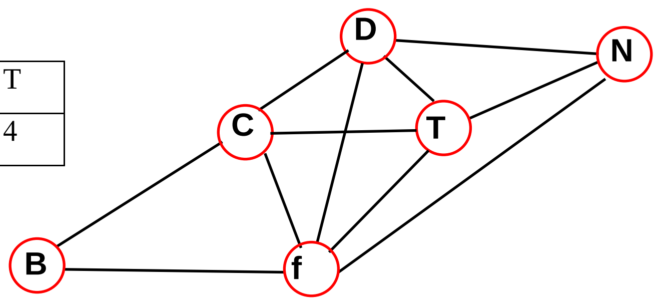
Summit	Degree	Color
A	5	C1
C	5	C2
B	4	C2
D	4	C3
E	4	C1
G	4	C3
F	3	C1
H	3	C4

The minimum number of aquariums is 4 (= no. of colors)
Aqua1: {A, E}
Aqua2: {C, B}
Aqua3: {D, G, F}
Aqua4: {H}

Exercise 03

1. Filling the table

Vertices	B	C	D	F	N	T
Vertices' Degrees	2	4	4	5	3	4



Graph G

1. Is graph G connected?

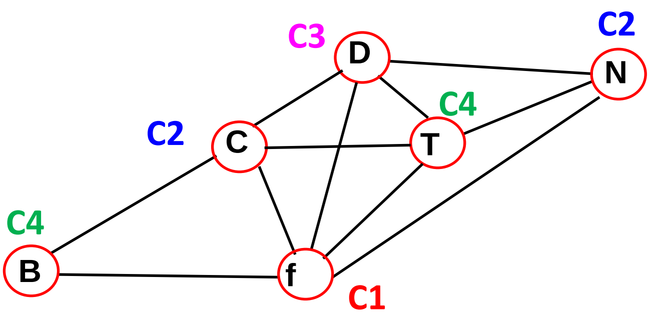
Yes the graph is connected, because each pair of vertices (x, y) is connected by a chain.

2. Yes, the group can pass through all six vertices by passing each path once and only once. To prove this, we need to look for an Eulerian chain in the graph G.
   And since the number of odd degree vertices is 2 (f, N) ==> an Eulerian chain in the graph G ==> for example: {F, B, C, F, N, T, F, D, C, T, D, N}

3. The group wants to associate each vertex with a color so that vertices connected by a path do not have the same color. (n is the chromatic number of the graph).
   a) Demonstration that: $4 \leq n \leq 6$
      The maximum degree of a vertex in graph G is d(f) = 5
      And according to the properties seen during this chapter, $n \leq dmax + 1$ ==> $n \leq 6$
      Moreover, the subgraph {C, F, D, T} of order 4, is the largest clique in G (the largest complete subgraph in G) ==> $n \geq 4$ (we need at least 04 colors to color it)
      So we get the following: $4 \leq n \leq 6$

   b) Proposal for a graph coloring by applying the Welch Powell algorithm

Summit	Degree	Color
F	5	C1
C	4	C2
D	4	C3
T	4	C4
N	3	C2
B	2	C4



Graph G

Good luck