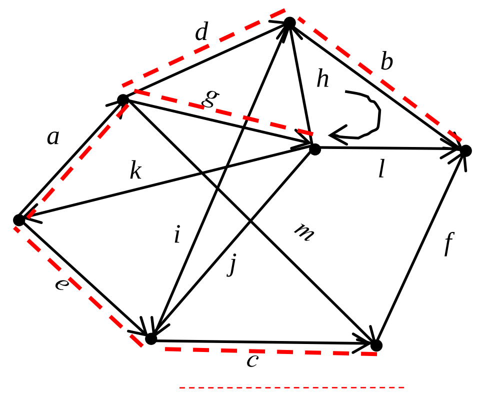
TUTORIALS - SERIES NO. 04 (CORRECTION)

Exercise 01

Let the following graph G(X, U):



1. The maximal tree T of G.

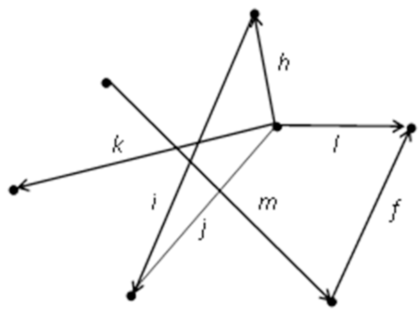
A maximal tree of G is a connected, cycle-free partial graph T of G of order n and size $n-1$.
The algorithm for constructing the tree T of the graph G consists of taking the $(n-1)$ edges that do not form cycles (We keep all the vertices and we delete edges that close (form) cycles)
We have the order of G is $n = 7$ and its size is $m = 13$ ==> the tree associated with G is then characterized by: Order $n = 7$, size m'= $n - 1 = 6$

Edge			*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*I*	*k*	*L*	*m*
Decision			ok	ok	ok	ok	ok	No	ok	No	No	No	No	No	No

The resulting maximal tree is plotted on the graph by ***a dashed red line***.

2. The co-tree T' associated with T.

The co-tree is the complementary graph of the tree T with respect to G, i.e., we eliminate from G all the edges that form the tree T to obtain the cotree T'



3. Vectors associated with cycles: C_b = (b,l,j,e,a,d); C_h = (h,d,a,e,j); C_i = (i,e,a,d); C_f = (f,l,j,e,a,m); C_c = (c,m,a,e); C_k = (k,e,j); C_g = (g,j,e,a)

Vectors → Cycles ↓	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*I*	*k*	*L*	*m*
C_b = (b,l,j,e,a,d) ==> V_{Cb} =	+1	+1	0	+1	-1	0	0	0	0	+1	0	-1	0
C_h = (h,d,a,e,j) ==> V_{Ch} =	+1	0	0	+1	-1	0	0	-1	0	+1	0	0	0
C_i = (i,e,a,d) ==> V_{Ci} =	+1	0	0	+1	-1	0	0	0	+1	0	0	0	0
C_f = (f,l,j,e,a,m) ==> V_{Cf} =	+1	0	0	0	-1	-1	0	0	0	+1	0	-1	+1
C_c = (c,m,a,e) ==> V_{Cc} =	+1	0	-1	0	-1	0	0	0	0	0	0	0	+1
C_k = (k,e,j) ==> V_{Ck} =	0	0	0	0	-1	0	0	0	0	+1	+1	0	0
C_g = (g,j,e,a) ==> V_{Cg} =	+1	0	0	0	-1	0	+1	0	0	+1	0	0	0

4. Calculation of the cyclomatic number V(G)

We calculate the cyclomatic number which is the dimension of the cycle base

$$V(G) = m - n + p = 13 - 7 + 1 = 7$$

i.e., our cycle base, if it exists, is composed of 07 independent cycles

5. Proposal of a cycle base for the graph G.

To construct this base, we take the tree T and the cotree T' ==> the arcs which form unique cycles in T are the arcs of the co-tree (i, k, m, h, l, j, f)

So, the cycles c_i = (a, d, **i**, e), c_k = (a, g, **k**), c_m = (a, **m**, c, l), c_h = (g, **h**, d), c_l = (d, b, **l**, g), c_j = (a, g, **j**, e), c_f = (a, d, b, **f**, c, e) are independent because each one contains an arc that the others do not contain ==> form a base of cycles

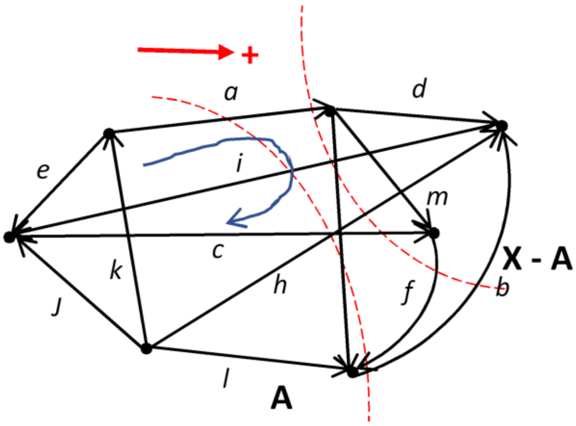
Similarly, we see that cycles given in (3) can constitute a cycle base because:

- Their number is 7
- They are independent, because every cycle contains an edge that the others do not contain.

Therefore, the cycles C_b = (b,l,j,e,a,d); C_h = (h,d,a,e,j); C_i = (i,e,a,d); C_f = (f,l,j,e,a,m); C_c = (c,m,a,e); C_k = (k,e,j); C_g = (g,j,e,a) constitute a cycle basis for the graph G.

Exercise 02

Let the following graph G(X, U):



1. Vectors associated with cocycles:

θ_a = (a,b,c,f,g,h,i); θ_d = (d,b,h,i); θ_e = (e,b,c,f,g,h,i,k);

θ_j = (j,b,f,g,h,k); θl = (l,b,f) ; θ_m = (m,c,f).

Vectors → Cocycles ↓	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*I*	*k*	*L*	*m*
θa = (a,b,c,f,g,h,i) ==> $V_{\theta a}$ =	+1	+1	+1	0	0	-1	-1	+1	-1	0	0	0	0
θ_d = (d,b,h,i)==> $V_{\theta d}$ =	0	+1	0	0	0	0	0	+1	-1	0	0	0	0
θ_e = (e,b,c,f,g,h,i,k) ==>$V_{\theta e}$ =	0	+1	+1	0	0	-1	-1	+1	-1	0	0	0	0
θ_j = (j,b,f,g,h,k)==> $V_{\theta j}$ =	0	+1	0	0	0	-1	-1	+1	0	0	0	0	0
θ_l = (l,b,f) ==> $V_{\theta l}$ =	0	+1	0	0	0	-1	0	0	0	0	0	0	0
θ_m = (m,c,f). ==> $V_{\theta m}$ =	0	0	+1	0	0	-1	0	0	0	0	0	0	0

2. Calculation of the cocyclomatic number $\lambda(G)$.

We calculate the cocyclomatic number which is the dimension of the cocycle base

$$\lambda(G) = n - 1 = 7 - 1 = 6$$

i.e. our cocycle base is composed of 06 independent cocycles

3. Proposal of a cocycle basis for the graph G

We note that the cocycles given in (1) can constitute a base of cocycles since:

- Their number is 6
- They are independent, because every cocycle contains an edge that the others do not contain

So, the cocycles: $\theta_a = (a,b,c,f,g,h,i)$; $\theta_d = (d,b,h,i)$; $\theta_e = (e,b,c,f,g,h,i,k)$; $\theta_j = (j,b,f,g,h,k)$; $\theta_l = (l,b,f)$; $\theta_m = (m,c,f)$ constitute a cocycle basis for the graph G.

4. Verifying the relationship: $\vec{V}_{ci}.\overrightarrow{V_{\theta i}}$= **0**

Let us take a cycle and a cocycle arbitrarily, for example:

$c_i = (e, a, g, l, j)$; $\theta_i = (a,b,c,f,g,h,i)$

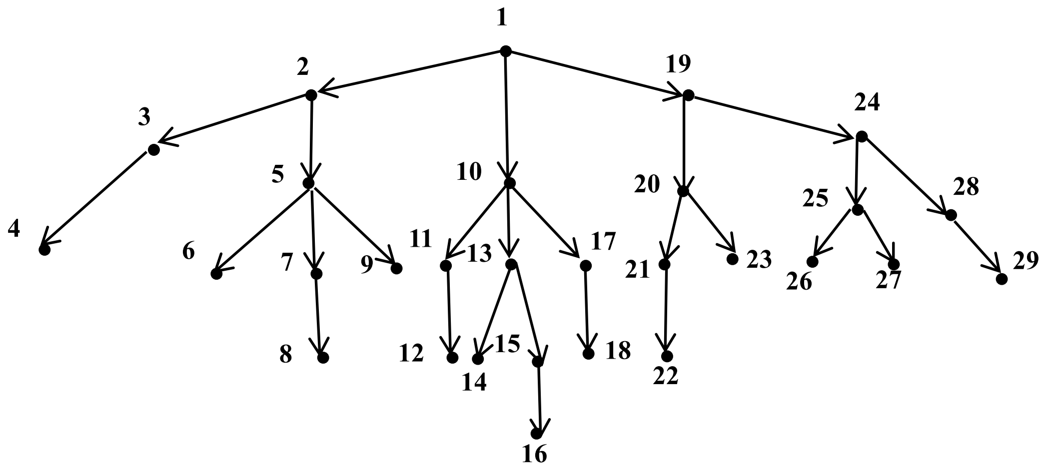
We look for their associated vectors and then calculate the scalar product $V_{ci}.\ V_{\theta i}$ as shown in the table below:

	a	b	c	d	e	f	g	h	i	I	k	L	m	
c = (e, a, g, l, j)	+1	0	0	0	-1	0	+1	0	0	+1	0	-1	0	
θa = (a,b,c,f,g,h,i) ➔vθa =	+1	+1	+1	0	0	-1	-1	+1	+1	0	0	0	0	
$\vec{V}_{ci}.\overrightarrow{V_{\theta i}}$	+1	0	0	0	0	0	-1	0	0	0	0	0	0	**0**

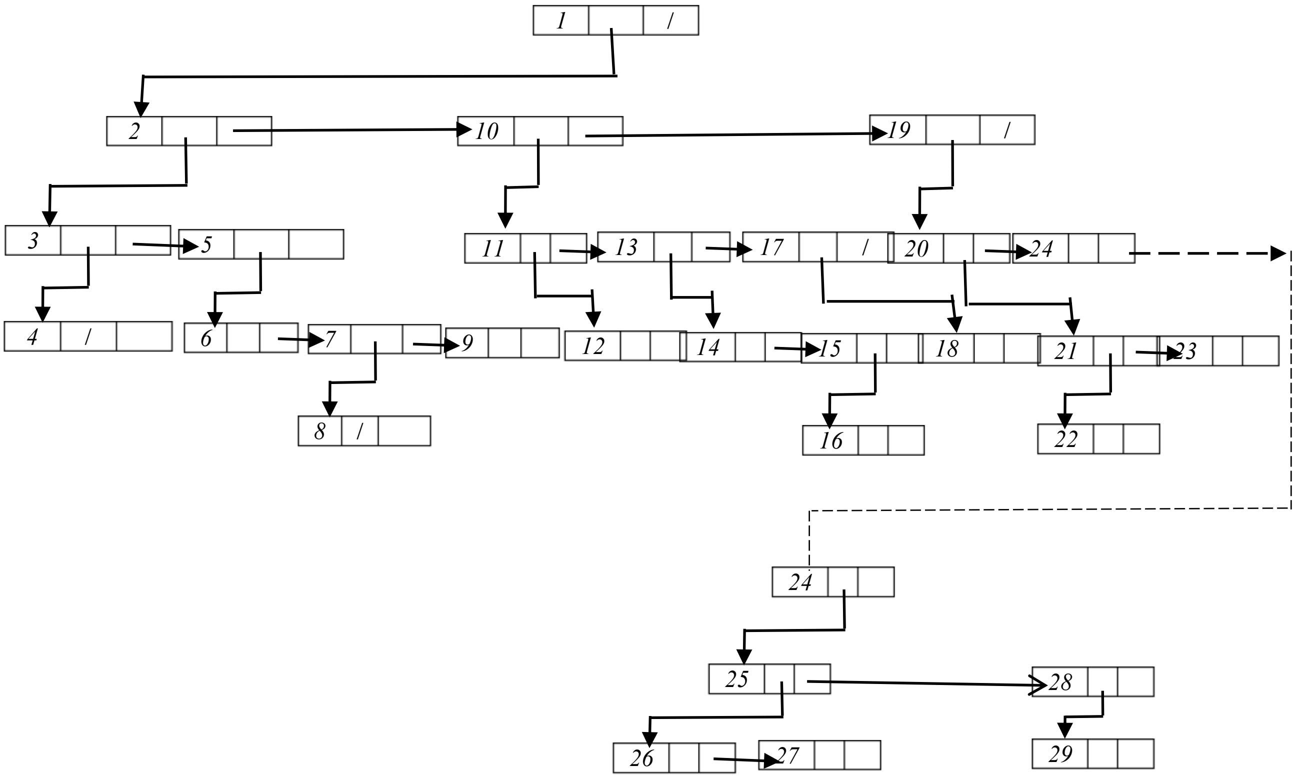
We can generalize this for all vectors of cycles and cocycles.

<u>Exercise 03</u>

Consider the following arboresence:



1. A representation for this arboresence.



2. Width browsing of the arboresence.

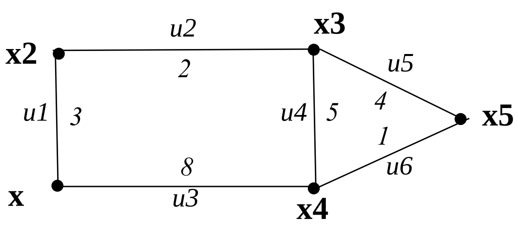
It is done in the following order:
{1, 2, 10, 19, 3, 5, 11, 13, 17, 20, 24, 4, 6, 7, 9, 12, 14, 15, 18, 21, 23, 25, 28, 8, 16 , 22, 26, 27, 29}

3. Depth browsing of the arboresence.

It is done in the following order:
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25 , 26, 27, 28, 29}

<u>Exercise 04</u>

Let G(X, U, W) be a connected valued graph



Find the maximum tree of minimum weight using:

a) – KRUSKAL 1 algorithm

Application of the algorithm:

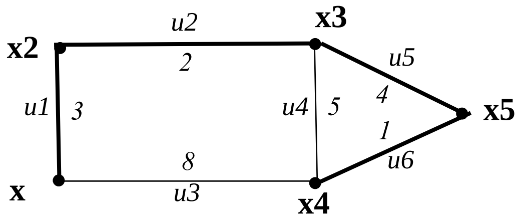
Unsorted edge list:

Edge	u_1	u_2	u_3	u_4	u_5	u_6
Weight	3	2	8	5	4	1

After sorting we will have:

List of edges sorted by their weights

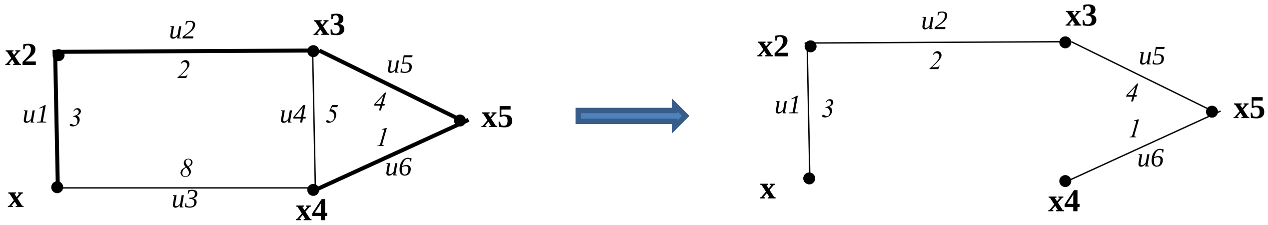
Edge	u_6	u_2	u_1	u_5	u_4	u_3
Weight	1	2	3	4	5	8



Search for the maximal tree of minimum weight (order n=5, size m'=n-1=5-1=4)

Edge	*u6*	*u2*	*u1*	*u5*	*u4*	*u3*
Weight	1	2	3	4	5	8
Decision	ok	ok	ok	ok	No	No

Total weight of maximum tree minimum weight = 1+2+3+4 = 10



b) – KRUSKAL 2 algorithm

Application of the algorithm:
We take the unsorted list of edges

Edge	*u1*	*u2*	*u3*	*u4*	*u5*	*u6*
Weight	3	2	8	5	4	1
Decision. Prov.	ok	ok	ok	ok	ok	ok
Final Decision	ok	ok	**No**	**No**	ok	ok

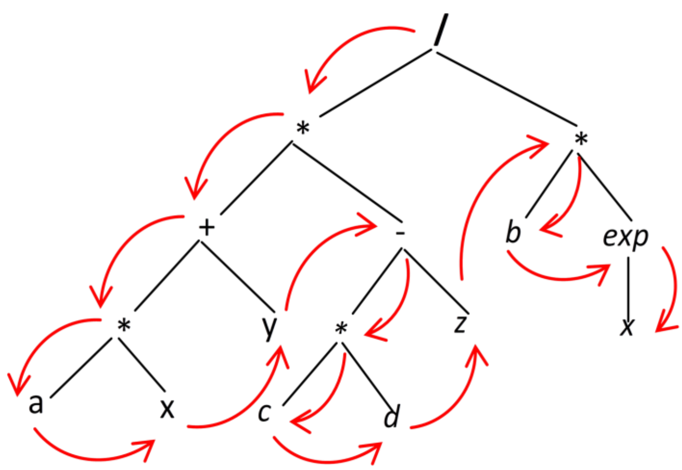
We obtain the same tree as in the previous figure: $\{u_1, u_2, u_5, u_6\}$ with a total weight = 10

Exercise 05

1. Representation of the arithmetic expression (ax + y)(cd – z) / (be^x) in the form of an arboresence respecting the rules of operator precedence.

(a*x + y)*(c*d – z) / (b*e^{x})

3 4 2 3 4 1 2 3 **Operator priority**



2. The expression in Polish notation.

Polish notation consists of always putting operators before operands, this notation is obtained by a depth browsing of the previous tree

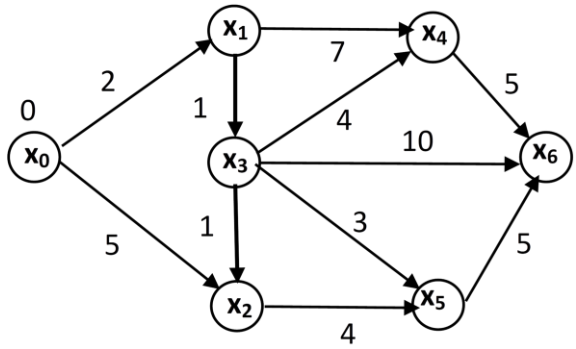
/* + * axy - * cdz * b exp x

Exercise 06

1. Find a shortest path SP Between x_0 and x_4
   There are only two paths C_1, C_2 between x_0 and x_4

   C1 = (x0, x1, x4) ==> w(C1) = 9
   C2 = (x0, x1, x3, x4) ==> w(C1) = 7   this is the SP

2. Application of the Moore-Dijkstra algorithm on the graph to find the SPs between x0 and all other vertices.



The Moore-Dijkstra algorithm

i) $S = \{s\}$;$\alpha = s$;$\pi(s) = \pi(\alpha) = 0$;$\pi(x) = +\infty \ \ \forall x \in (X - S)$; $A(x) = \phi \ \ \forall x \in (X - S)$

ii) We consider all arcs u such that: $I(u) = \alpha$ and $T(u) = x \in (X - S)$
If $\pi(\alpha)+d(u) < \pi(x)$ then set $\pi(x) = \pi(\alpha)+d(u)$; $A(x) = \{u\}$
Otherwise continue

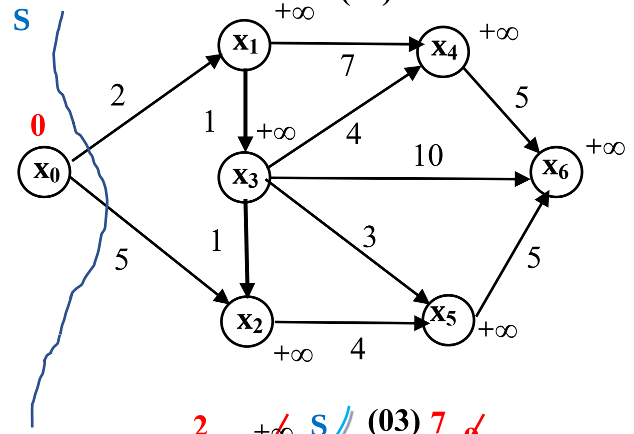
iii) Choose a vertex $y \in (X - S)$;$\pi(y) = \text{Min}[\pi(x)] \ \ \forall x \in (X - S)$
If $\pi(y) = +\infty$ then stop ==> S is not a root of G
Otherwise put$\alpha = y$ $S = S \cup \{\alpha\}$

If S = X then finish {problem is solved}
Otherwisego to (i)

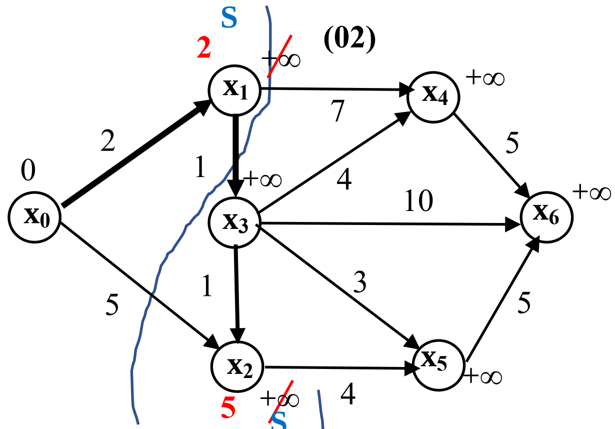
The progress of the algorithm is summarized in the table below and the associated graphs:

Iter No.	**Arcs u chosen**	**S**	**α**	**$\pi(\alpha)$**	**xi $\in$(X–S)**	**$\pi(x_i)$**	**A(xi)**	**y**	**graph**
0 (init)	none	$\{x_0\}$	x_0	0	$\{x_1, x_2, x_3, x_4, x_5, x_6\}$	$\{+\infty, +\infty, +\infty, +\infty, +\infty, +\infty\}$	None	None	(01)
1	(x_0, x_1) (x_0, x_2)	$\{x_0\}$ $\{x_0, x_1\}$	/ x_1	0 2	$\{x_1, x_2\}$ $\cup$ $\{x_3, x_4, x_5, x_6\}$	$\{0+2 = 2, 0+5\}$ $\{+\infty, +\infty, +\infty, +\infty\}$	(x_0, x_1), (x_0, x_2)	x_1	(02)
2	(x_1, x_3), (x_1, x_4)	$\{x_0, x_1\}$ $\{x_0, x_1, x_3\}$	/ x_3	3	$\{x_2, x_3, x_4\}$ $\cup$ $\{x_5, x_6\}$	$\{5, 2+1, 2+7\}$ $\{+\infty, +\infty\}$	(x_1, x_3) (x_1, x_4),	x_3	(03)
3	(x_3, x_2) (x_3, x_4) (x_3, x_5) (x_3, x_6)	$\{x_0, x_1, x_3\}$ $\{x_0, x_1, x_3, x_2\}$	/ x_2	4	$\{x_2, x_4, x_5, x_6\}$	$\{3+1, 3+4, 3+3, 3+10\}$	(x_3, x_2), (x_3, x_4) (x_3, x_5), (x_3, x_6)	x_2	(04)
4	(x_2, x_5)	$\{x_0, x_1, x_3, x_2\}$ $\{x_0,x_1,x_3,x_2,x_5\}$	/ x_5	6	$\{x_4, x_5, x_6\}$	$\{7, 6, 13\}$	(x_2, x_5)	x_5	(05)
5	(x_5, x_6)	$\{x_0,x_1,x_3,x_2,x_5\}$ $\{x_0,x_1,x_3,x_2,x_5,x_4\}$	/ x_4	7	$\{x_6\}$	$\{6+5=11\}$	(x_5, x_6)	x_4	(06)
6	(x_4, x_6)	$\{x_0,x_1,x_3,x_2,x_5,x_4\}$ $\{x_0,x_1,x_3,x_2,x_5,x_4, x_6\}$	/ x_6	11			(x_4, x_6)	x_6	(07)
7	none	{x0,x1,x3,x2,x5,x4,x6}	S = X ==> Algorithm completed						(08)

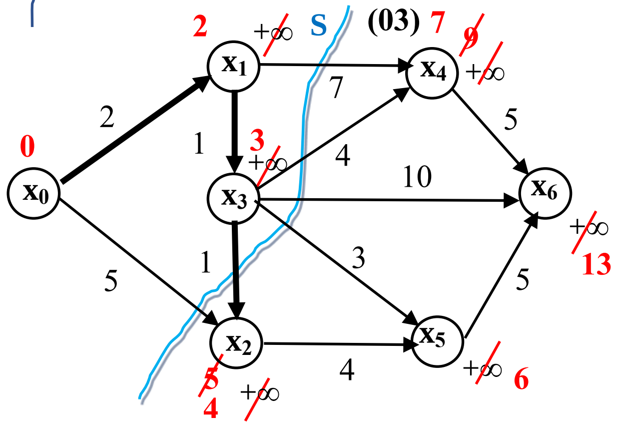
(01)



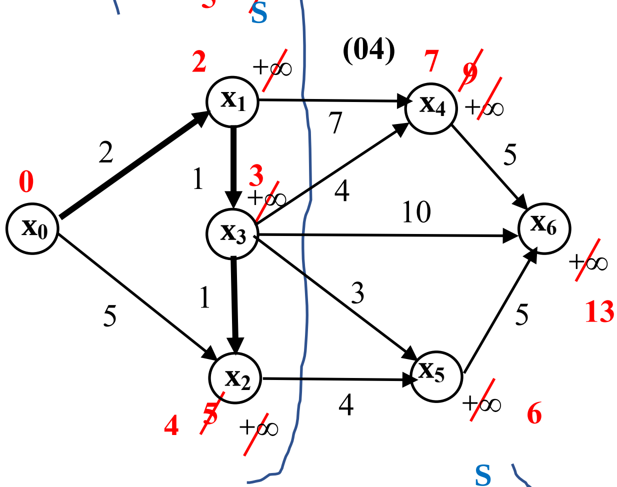
(02)



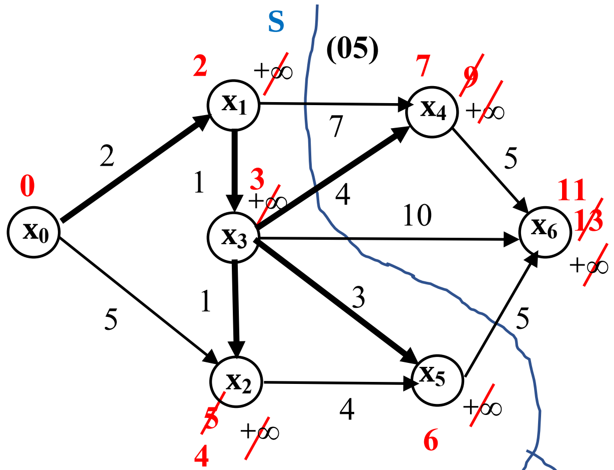
(03)



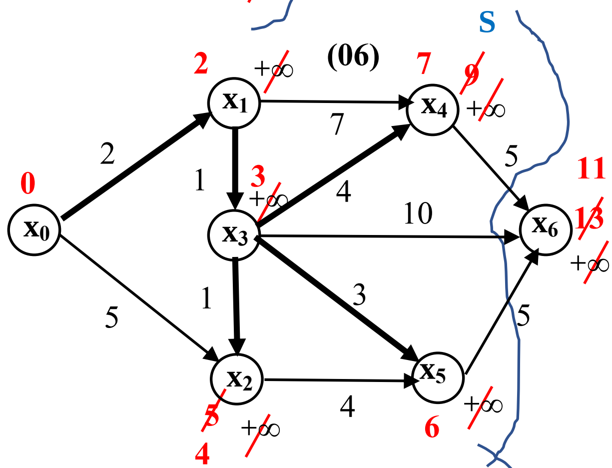
(04)



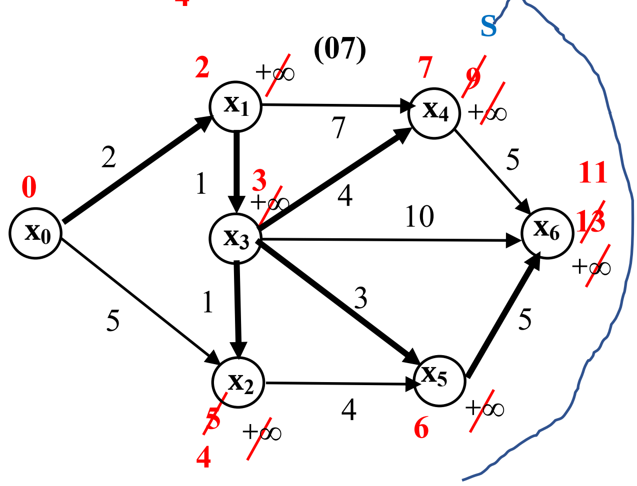
(05)



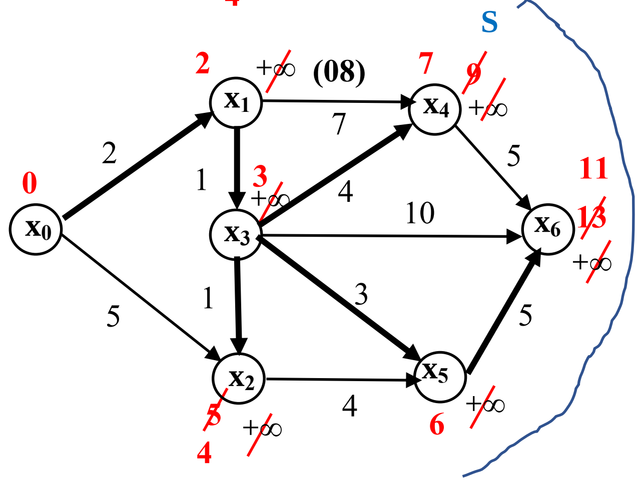
(06)



(07)

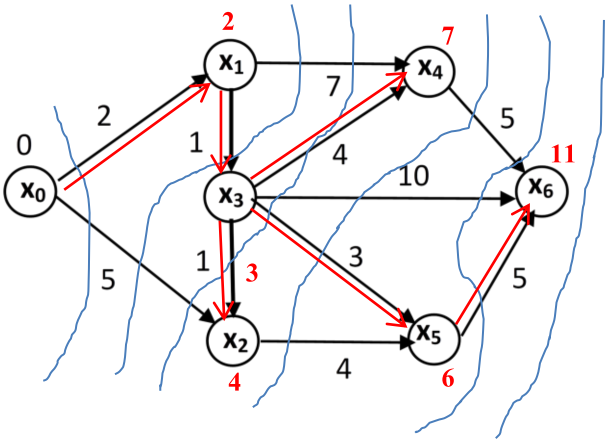


(08)



OBS: the SPs between x0 and all other paths form a tree.

3. Application of Bellman's algorithm on the same previous graph



4. The SPs between x0 and the vertices: x4, x5, x6
   According to the last graph (08):
   SP(x0, x4) = {x0, x1, x3, x4} ==> w(x0, x4) = 7
   SP(x0, x5) = {x0, x1, x3, x5} ==> w(x0, x5) = 6
   SP(x0, x6) = {x0, x1, x3, x5, x6} ==> w(x0, x6) = 11