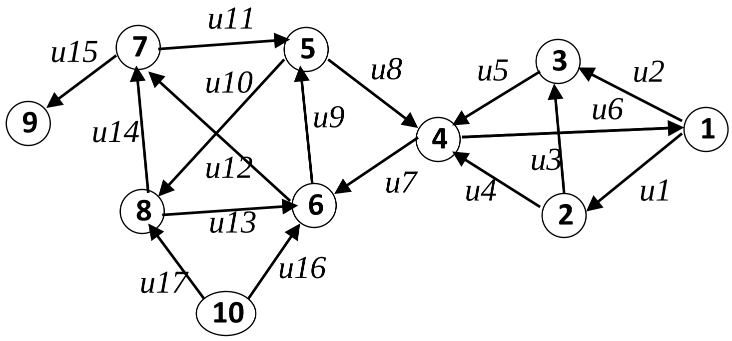
TUTORIALS - SERIES NO. 01 CORRECTED

Exercise 01

1. Graph order: $n = |X| = 10$

   Graph size: $m = |U| = 17$



2. Representation of the graph using:

a) An adjacency matrix (vertex-vertex)

	1	2	3	4	5	6	7	8	9	10
1	0	1	1	0	0	0	0	0	0	0
2	0	0	1	1	0	0	0	0	0	0
3	0	0	0	1	0	0	0	0	0	0
4	1	0	0	0	0	1	0	0	0	0
5	0	0	0	1	0	0	0	1	0	0
6	0	0	0	0	1	0	1	0	0	0
7	0	0	0	0	1	0	0	0	1	0
8	0	0	0	0	0	1	1	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	1	0	1	0	0

b) An incidence matrix (vertex – arcs)

	u1	u2	u3	u4	u5	u6	u7	u8	u9	u10	u11	u12	u13	u14	u15	u16	u17
1	+1	+1	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0
2	-1	0	+1	+1	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	-1	-1	0	+1	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	-1	-1	+1	+1	-1	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	+1	-1	+1	-1	0	0	0	0	0	0
6	0	0	0	0	0	0	-1	0	+1	0	0	+1	-1	0	0	-1	0
7	0	0	0	0	0	0	0	0	0	0	+1	-1	0	-1	+1	0	0
8	0	0	0	0	0	0	0	0	0	-1	0	0	+1	+1	0	0	-1
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	+1

3. $\Gamma^{+}(6) = \{5, 7\}$,$\Gamma^{-}(4)$, = $\{2, 3, 5\}$, $\Gamma(5) = \{4, 6, 7, 8\}$.
4. $d+(8) = 2$, $d^{-}(2) = 1$, $d(6) = 5$.
5. $I^{+}(4) = \{u_6, u_7\}$, $I^{-}(6) = \{u_7, u_{13}, u_{16}\}$, $I(8) = \{u_{10}, u_{13}, u_{14}, u_{17}\}$.

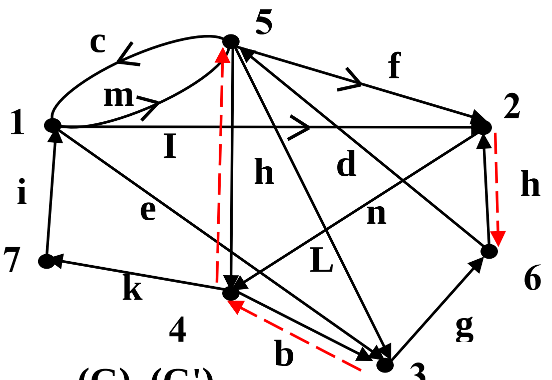
6. Find a path between vertices 4 and 6 ? is it simple? Is it elementary?
   C1 = (4, 6) simple and elementary path
   C2 = (4, 6, 7, 5, 8, 6) simple but not elementary path.

7. Give a simply connected component
   Simple CC 1 = {1, 2, 3, 4}; CSC 2 = {5, 6, 7, 8, 9, 10}

8. Give a strongly connected component
   Strong CC 1 = {1, 2, 3, 4}; CFC 2 = {4, 5, 6, 7, 8}

Exercise 02

We have the graph G(X,U)
with:
X = {1, 2, 3, 4, 5, 6, 7}
U={a, b, c, d, e, f, g, h, i, j, k, l, m, n}



(G), (G')

1 - Among the n_uples below we specify: the paths, the simple paths, the elementary paths:

(g, d, f, i): is not a path (non-continuous path)
(h, b, e, j): is not a path (non-continuous path)
(b, g, d, c, e, g, a): a path, is not simple, is not elementary
(i, e, g, d, c, j): a simple and elementary path
(k, i, e, g, d): a simple and elementary path

2 – Yes, the two paths (5, 1, 3, 6, 5, 2) and (c, e, g, d, f) are identical

3 - The path (4, 7, 1, 3, 6, 5, 4, 3, 6, 5, 4) is a circuit, but it is not simple, nor elementary?

4- The path (3, 6, 2, 4, 7, 1, 5, 4, 3): is a circuit, simple, not elementary.
The path (5, 4, 7, 1, 3, 6, 5): is a circuit, simple and elementary.

After reversing the arcs: a, b, h, d, we will have:

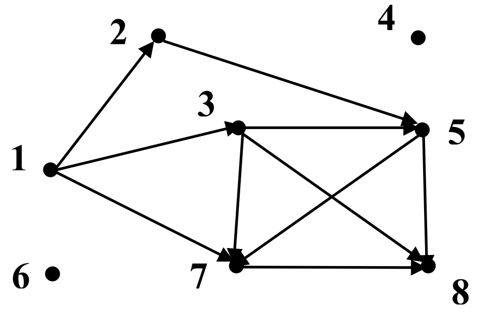
5 - (b, k, i, m, f, a): a Hamiltonian path (elementary path that passes through all the vertices of the graph)
(j, n, h, c, m, l, b, k, i, e, g, d, f, a): an Eulerian path

6 – The two previous paths can be expressed by the following sequences of vertices:
(b, k, i, m, f, a) <==> (3, 4, 7, 1, 5, 2, 6),
(j, n, h, c, m, l, b, k, i, e, g, d, f, a) <==> (1,2,4, 5, 1, 5, 3, 4, 7, 1, 3, 6, 5, 2, 6).

7 – Yes, the circuit (k, i, j, a, d, l, b) is a Hamiltonian circuit
Its equivalent in sequence of vertices is: (4, 7, 1, 2, 6, 5, 3, 4)

Exercise 03

Let G(X,U) be a directed graph



1 – A source vertex: {1}, there is also: {4, 6},
 A sink vertex: {8}, there is also: {4, 6}
 isolated vertices : {4, 6}

2 - $d^{+}(4) = 0$, $\Gamma(4) = \phi$, $d(1) = 3$, $\Gamma(1) = \{2, 3, 7\}$
 $d^{-}(6) = 0$,$\Gamma^{+}(1) = \{2, 3, 7\}$,$\Gamma^{-}(7) = \{1, 3, 5\}$, $d(3) = 4$

3 – The order and size of the graph G.
 Order $n = |X| = 8$, Size $m = |U| = 10$

4 – A simple path between 1 and 8: (1, 3, 8), (1, 7, 8), (1, 2, 5, 7, 8}

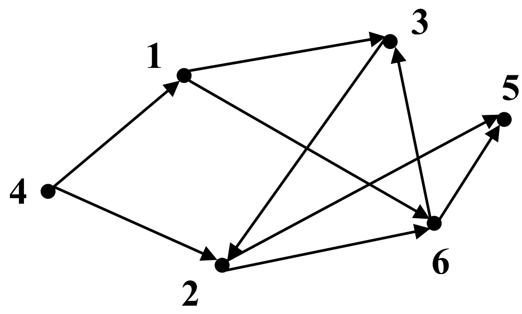
5 – A chain between 2 and 7: (2, 1, 7), (2, 5, 3, 1, 7)

Exercise 04

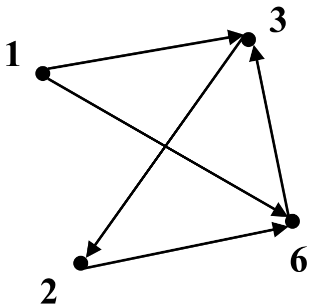
Let the following graph G(X, U):

With: X = {1, 2, 3, 4, 5, 6}

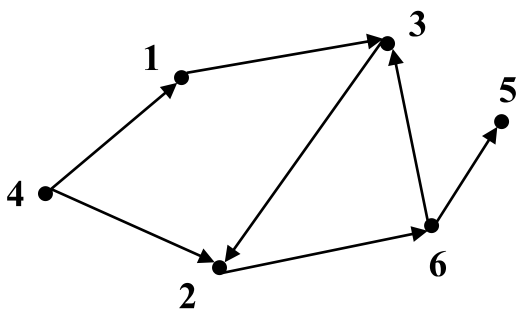
U = {(4.2), (2, 6), (1.3), (6.5), (1.6), (6.3),
 (2.5), (3.2), (4.1)}



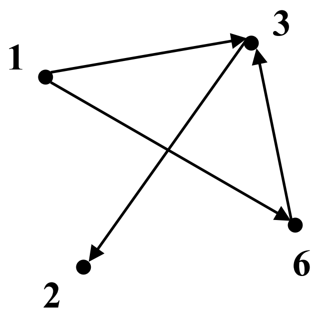
Which of the following graphs is a: subgraph, a partial graph, a partial subgraph?



(G1)



(G2)



(G3)

(G1): a subgraph of G (we have retained some vertices and some arcs)
(G2): a partial graph of G (we took all the vertices, but some arcs)
(G3): a partial subgraph of G (a partial graph of the subgraph G1)