Faculty of Mathématics and Informatics	Date : 14.01.2026	Level : 2nd Year Licence LMD
Department of Computer Science	Duration: 02h	Course: Graph Theory

Exam of Semester 3

EXERCISE 01 — Course Questions (20 Minutes) — 03 points

1. What do you know about the following graphs : ***(02 pts)***
   (a) A simple graph
   (b) A complete graph
   (c) A complementary graph
   (d) A planar graph

2. Let G(X, U) be an undirected graphe of order ***n*** and size **m**, x_i is a vertex of G, k_i is the degree of x_i, a_i is an element of the adjacency matrix ***M*** representing G, ***L(L = m)*** is the the size of G, give the formula of ***L*** in terms of : k_i, a_i. ***(01 pt)***

EXERCISE 02 — 20 Minutes — 04 points

A football league consists of 7 teams. For time reasons, it is decided that each team will play only 3 matches.

1. Is this possible? Justify your answer. ***(01 pt)***
2. What is the minimum number of matches each team must play for this to be possible? Justify. ***(01 pt)***

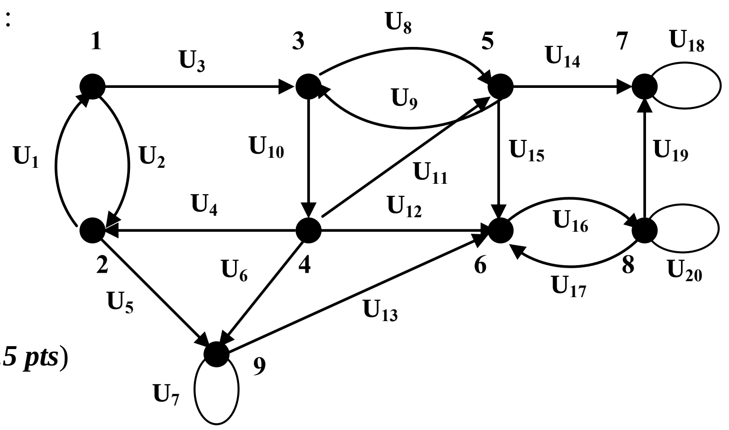
Now suppose we have 10 teams, and each team must play 3 matches.

- Answer the same questions (1) and (2) ***(01 + 01 pts)***

EXERCISE 03 — 30 Minutes — 05 points

Let the following graphe G(X, U) defined as follows :

With : $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$U = \{u_1, u_2, \ldots\ldots, u_{20}\}$



1 . Find in graph G : ***(1.5 pts)***
- A simple path between vertices 1 and 8.
- Elementary path between vertices 3 and 9.
- A hamiltonian path.

2. Find : $\Gamma^-(9)$, $d^+(4)$, $\Gamma(5)$, $\Gamma(8)$ (***01 pt***)

3 . Represent the graph G using an adjacency matrix (***1.5 pts***)

4. Find the strongly connected components SCCs by applying the marking algorithm. (***01 pt***)

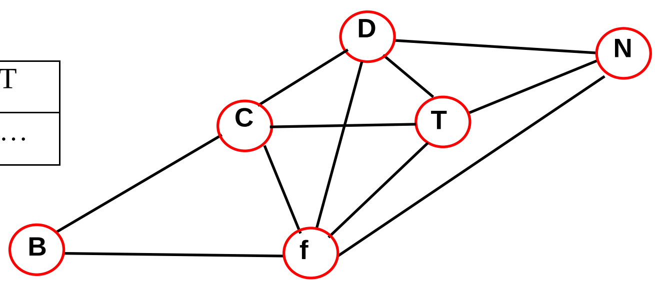
EXERCICE 04 — 25 Minutes — 5.0 points

A group of friends is organizing a walk in the mountains. The graph below represents the vertices B, C, D, F, T, N through which they can choose to pass.

An edge between two vertices indicates the existence of a path between those two vertices.

1. Complete the following table: **(1.5 pts)**

Vertices	B	C	D	F	N	T
Vertex degree	…	…	…	…	…	…



Graphe G

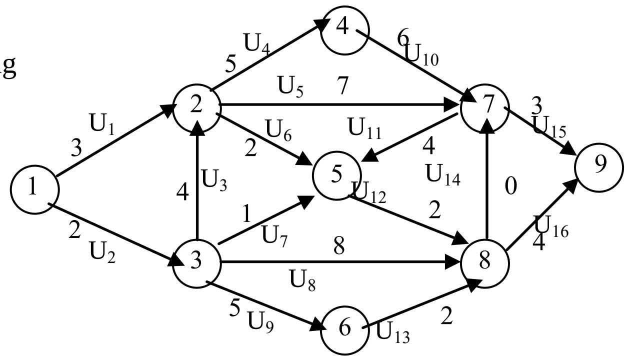
The group wishes to visit all six vertices by passing through each path exactly once.

2. Prove that this is possible and give an example of a route the group could take. **(1.5 pts)**
3. The group wishes to assign a color to each vertex such that vertices connected by a path do not have the same color. Let n be the chromatic number of the graph.
   a) Show that : $4 \le n \le 6$ **(01 pt)**
   b) Propose a coloring of the graph that allows the determination of its chromatic number.. **(01 pts)**

EXERCISE 05	**25 Minutes**	**04 points**

Let be the following graph G(X, U, C) :

1. Construct the maximal tree T with the minimum weight using KRUSKAL 1 algorithm ***(1.5 pts)***
2. Represent the tree T and the co-tree T ' on the graph G (use 2 different colors) ***(01 pt)***
3. Calculate the cyclomatic number V(G) ***(0.5 pt)***
4. Propose a cycle base for G ***(01 pt)***



Good Luck
Dr. Said Gadri

Faculty of Mathématics and Informatics	Date : 14.01.2026	Level : 2nd Year Licence LMD
Department of Computer Science	Duration: 02h	Course: Graph Theory

Exam of Semester 3 : Standard Correction

EXERCISE 01	Course Questions (20 Minutes)	03 points

1. What do you know about the following graphs : ***(02 pts)***

(a) A simple graph : it is a graph caracterized by :
- Doesn't contain any loop
- $\forall x, y \in X, \exists$at most $u=(x, y) \in U$

(b) A complete graph : it is a graph caracterized by :
- $\forall x, y \in X$, $(x, y) \in U$ and $(y, x) \in U$

(c) A complementary graph : it is a graph caracterized by : The complementary graph of the graph G(X, U) is the graph G*(X, U*) such that:
- $(x, y) \in U \Longrightarrow (x, y) \notin U^*$

(d) A planar graph : if it is possible to draw it in the plane in such a way that any two arcs do not intersect except at their ends the graph is said planar

2. Let G(X, U) be an undirected graphe of order ***n*** and size **m**, x_i is a vertex of G, k_i is the degree of x_i, a_i is an element of the adjacency matrix ***M*** representing G, ***L(L = m)*** is the the size of G, give the formula of ***L*** in terms of : k_i, a_i.

The formula is as follos : ***(01 pt)***

$$L = \frac{1}{2}\sum_{i=1}^{N} k_i = \frac{1}{2}\sum_{ij}^{N} A_{ij}$$

OBS : According the formula, L is an even number

EXERCISE 02	20 Minutes	04 points

A football league consists of 7 teams. For time reasons, it is decided that each team will play only 3 matches.

1. Is this possible? Justify your answer. ***(01 pt)***

We model this problem using a graph, by representing the 7 teams as 7 vertices, and the degree of each vertex as the number of matches played by each team.

This means that the size of the graph is:

$$L=7\times3=21$$

According to ***Exercise 1, Question 2***, this is not possible, because the size of an undirected graph is always even.

2. What is the minimum number of matches each team must play for this to be possible? Justify. ***(01 pt)***

The minimum number of matches cannot be 1 for the same reason as above:

$L=7\times1=7$ which is not an even number.

Therefore, the minimum number of matches each team must play is 2:

$7\times2=14$ which is an even number.

Now suppose we have 10 teams, and each team must play 3 matches. Answer the same questions (1) and (2).

1. Yes, this is possible, because: ***(01 pt)***

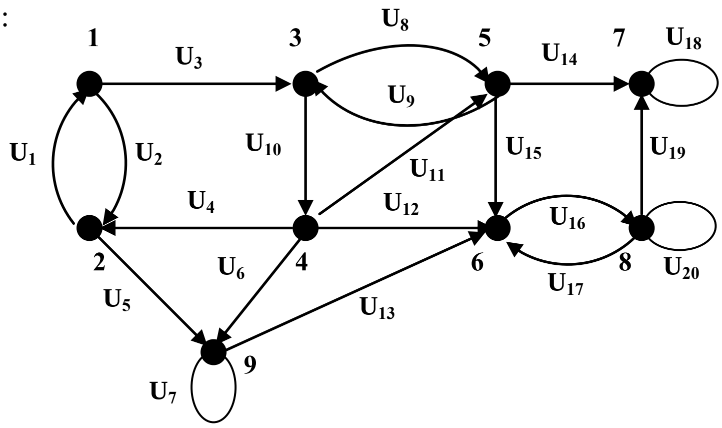
$L=10\times3=30$ which is an even number.

2. The theoretical minimum number of matches to play is 1, because: ***(01 pt)***

10×1=10 which is an even number (although this may be unacceptable according to football rules).

EXERCISE 03	**30 Minutes**	**05 points**

Let the following graphe G(X, U) defined as follows :



With : X = {1, 2, 3, 4, 5, 6, 7, 8, 9}
U = {u_1, u_2 , ……. , u_{20}}

1 . **Find in graph G** : ***(1.5 pts)***
- A simple path between vertices 1 and 8.
  SP(1, 8) = (1, 3, 4, 6, 8)
- Elementary path between vertices 3 and 9.
  EP = (3, 4, 9)
- A hamiltonian path.
  HP = (5, 3, 1, 2, 4, 9, 6, 8, 7)

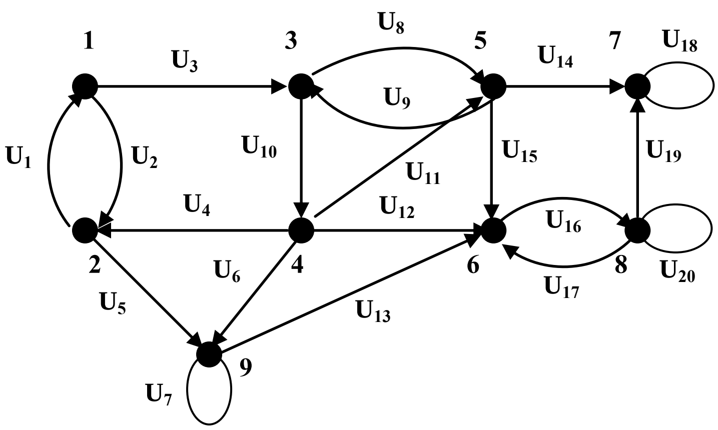
2. Find : $\Gamma^-(9)$, $d^+(4)$, $\Gamma(5)$, $I^-(8)$ (***01 pt***)
$\Gamma^-(9) = \{2, 4, 9\}$, $d^+(4) = 4$, $\Gamma(5) = \{3, 4, 6, 7\}$, $I^-(8) = \{U_{16}, U_{20}\}$

3 . **Represent the graph G using an adjacency matrix** (***1.5 pts***)

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	0	1	1	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	1
3	0	0	0	1	1	0	0	0	0
4	0	1	0	0	1	1	0	0	1
5	0	0	1	0	0	1	1	0	0
6	0	0	0	0	0	0	0	1	0
7	0	0	0	0	0	0	1	0	0
8	0	0	0	0	0	1	1	1	0
9	0	0	0	0	0	1	0	0	1

4. **Find the strongly connected components SCCs** by applying the marking algorithm. (***01 pt***)
There exist 04 components, which are :

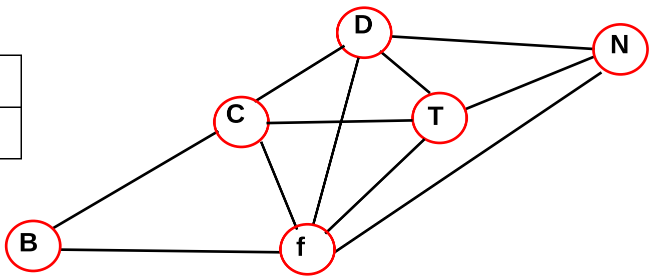
C1 = {1, 2, 3, 4, 5}, C2 = {6, 8}, C3 = {7}, C4 = {9}



EXERCISE 04	**15 Minutes**	**5.0 points**

1. Filling the table : ***(1.5 pts)***

Vertices	B	C	D	F	N	T
Vertex degree	2	4	4	5	3	4



Graphe G

2. Yes, the group can pass through the six vertices by using each path exactly once. ***(1.5 pts)***
To prove this, we must look for an Eulerian chain in graph *G*
Since the number of vertices with odd degree is 2 *(f, N)* ==> there exists an Eulerian chain in the graph G
==> for example, one possible route is : *{F, B, C, F, N, T, F, D, C, T, D, N}*

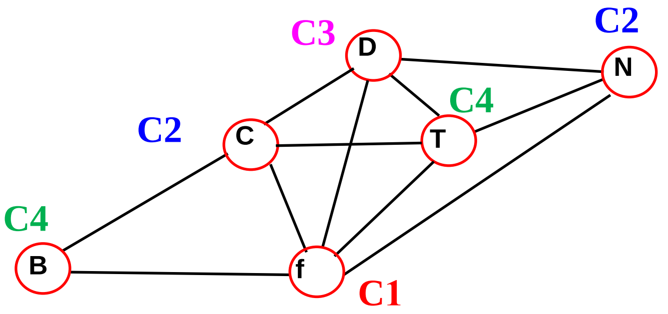
The group wishes to assign a color to each vertex such that vertices connected by an edge do not share the same color (***n denotes the chromatic number of the graph***).

a) Proof that : $4 \leq n \leq 6$ ***(01 pt)***
The maximum degree of a vertex in graph G is ***d(f) = 5***
According to the properties studied in this chapter, $\boldsymbol{n \leq dmax + 1}$ ==> $n \leq 6$
Moreover, the subgraph{C, F, D, T} of order 4 is the largest clique in *G* (the largest complete subgraph of G) ==> $\boldsymbol{n \geq 4}$ (we need to have at least 04 colors for coloring)

Hence, we obtain the following: $\boldsymbol{4 \leq n \leq 6}$

b) Graph coloring using the Welch–Powell algorithm ***(01 pts)***

Vertex	Degree	Color
F	5	C1
C	4	C2
D	4	C3
T	4	C4
N	3	C2
B	2	C4



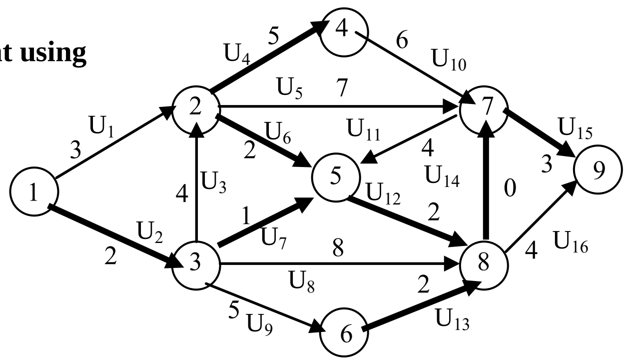
Graphe G

n = 4 colors **(**$\boldsymbol{4 \leq n \leq 6}$**)**

EXERCISE 05	**30 Minutes**	**4.0 points**

Let be the following graph G(X, U, C) :

1. Construct the maximal tree T with the minimum weight using KRUSKAL 1 algorithm *(1.5 pts)*



G is charcterized by :
The order n = |X| = 9, m = |U| = 16

➔ The obtained tree is characterized by :
The order n' = n = 9, m' = n-1 = 9-1 = 8 ➔

List of not sorted arcs

Arcs	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16
Weights	3	2	4	5	7	2	1	8	5	6	4	2	2	0	3	4
Decision																

We want to construct the maximal tree with minimum weight ==> We must sort arcs in the decreasing order of weights

Arcs	U14	U7	U2	U6	U12	U13	U1	U15	U3	U11	U16	U4	U9	U10	U5	U8
Weights	0	1	2	2	2	2	3	3	4	4	4	5	5	6	7	8
Decision	OK	OK	OK	OK	OK	OK	X	OK	X	X	X	OK	X	X	X	X

T_{max} = {U14, U7, U2, U6, U12, U13, U15, U4} $P(T_{max}) = 17$

2. Represent the tree T and the co-tree T ' on the graph G.
(use 2 different colors) ***(01 pt)***

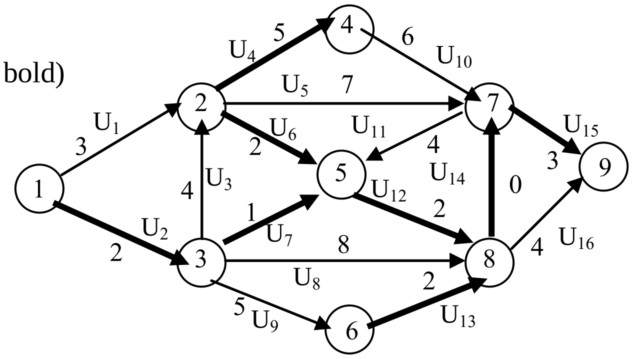
- The maximal tree is represented in the graph by the bold police
- The co-tree is represented by simple lines

3. Calculate the cyclomatic number V(G) ***(0.5 pt)***

V(G) = m-n+p = 16 – 9 + 1 = 8 ===> Dim(BC) = V(G) = 8

4. Propose a base of cycles for G ***(01 pt)***

We can deduce the BC by merging the maximal tree (in bold) and the co-tree (in simple lines)



==> BC = {C1 = (**U1**, U2, U6, U7), C3 = (**U3**, U6, U7), C5 = (**U5**, U6, U12, U14), C8 = (**U8**, U7, U12),
C9 = (U7, **U9**, U12, U13), C10 = (U4, U6, **U10**, U12, U14), U11 = (**U11**, U12, U14)
C16 = (U14, U15, **U16**)}

Good Luck
Dr. Said Gadri