Brief Presentation

1. Foreword

- We often hear people say: "a single picture is more meaningful than a thousand words"➔It is easier to represent a real problem using an image than to describe it using texts (usually long and carrying different interpretations that depend on the readers themselves).
  - ✓ A plan or 3D model for a house
  - ✓ An electronic circuit
  - ✓ A caricature for a social or political event
  - ✓ A geographical map of a country
  - ✓ A train map, tram or metro railway lines,
  - ✓ An organization chart of a company, etc.

The previous examples are based mainly on images or more precisely graphs.

- Graph theory is a very important field of knowledge today, because it offers key concepts to model more or less complex real problems and make them clearer and more understandable.

2. Birth and evolution

- In 1736 with the appearance of a communication made by the Swiss mathematician and physicist Leonhard Euler (1707 – 1783) in which he proposed a graphical solution to the famous problem of the seven (07) bridges of Königsberg which connect two islands on the Pregel River in Switzerland. The problem consisted of finding how to cross each of the bridges once and only once from a land on the banks of the river and return to the starting point. (Euler used in his solution vertices to represent lands and edges to represent bridges).

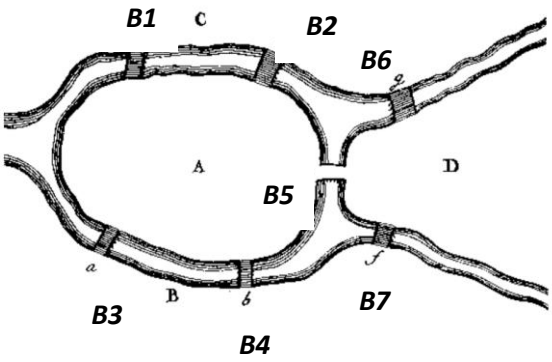

Figure 1. The seven (07) bridges of Königsberg

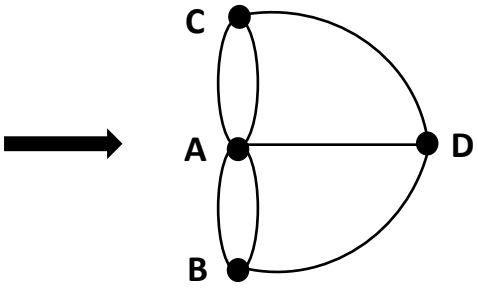

Figure 2. Solution proposed by Euler

- Since that date, graph theory had begun, gradually expanded, spread into most areas and became an indispensable tool for modeling and solving the majority of problems in our daily lives.

- A century after the appearance of Euler's problem, other problems were exposed, notably: the theory of trees used in the analysis of electrical circuits (Kirchhoff's Law 1824 – 1887).

- In the same period, Cayley (1821 – 1895) discovered the concept of the tree and used it in chemistry (to represent the saturated molecules of hydrocarbons of the type C_nH_{2n+2}.

- The four-color theory (four-color conjecture) which confirms that with four (04) colors one can color any flat map such as a country map, so that two neighboring countries do not have the same color (Möbus 1790 – 1868, Morgan (1806 – 1871) then became famous after its publication by Cayley in 1879 and found its solution in 1976 by Appel and Haken.

- Another problem proposed by Sir Hamilton (1805 – 1865) in 1959 was to find a path connecting 20 vertices representing cities placed on the heads of a regular dodecahedron (a polyhedron with 12 faces and 20 vertices), provided that this path does not pass through any of the vertices more than once. This problem is known by the name of its inventor (Hamiltonian path)

Figure 3. Dodecahedron in 3D

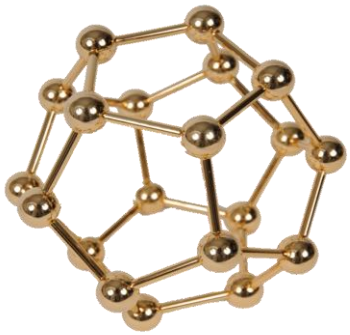

Figure 4. Dodecahedron in wireframe model

- A first work devoted entirely to graph theory was written by König in 1936, it is the first work which used the term graph for the first time.
- Since 1946, graph theory has undergone significant development and touched on concrete problems thanks to the efforts of researchers such as: Kuhn (1955), Ford and Fulkerson (1956), Claude Berge (1958), Roy (1959), etc.

- Since this work, graph theory has taken a very important place especially with the appearance of computers with their enormous graphical capabilities. This field is known by the term "operations research" or sometimes "discrete mathematics" and recently we hear the term "graph theory".

3. Areas of application

Graph theory is used in several fields and in countable applications for which the problems treated are mainly modeled using geometric structures that we call graphs, among these fields we can cite:

1. Chemistry: for the representation of atoms, molecules, and the links between them.
2. Architecture: for the design of house plans, buildings and cities.
3. Transportation: to represent road and rail networks, as well as to solve problems related to vehicle and pedestrian traffic.
4. Electrical networks: to represent intra and inter-city wiring, connections to homes, fault monitoring, etc.
5. Computer networks: either in a company (computers, connection cables, peripherals, information exchange), or even to consider the flow of information circulating on the global Internet network.
6. Telecommunications networks: in particular the placement of receiving antennas, access points, fault monitoring.
7. Hydraulics: to model AEP (DWS : Drinking Water Supply) networks as well as sanitation networks in towns or villages.
8. Electronics: circuit design (electronic card diagrams), representation of components, current path on the card.
9. Construction of large assemblies and structures: Aviation and aeronautics, production lines, movement of planets in astronomy, etc.
10. Markov chains: to consider the evolution of any system from one state to another, automata, etc.
11. Other areas