University of M'sila Faculty of MI Department of computer science	**Correction Exam** **(Normal Session)**	**Mathematical Logic (2nd level Bachelor)** **Academic year: 2025-2026** **Duration : 2h**

Exercise #1 (0.5 * 5 marks) :

Answer the following statements with **True** or **False** and justify the false one:

Affirmation	T/F	Why ?
We can translate "x + 1 = 2" sentence to an assertion in propositional logic.	F 0.25	non assertive statement 0.25
For a given formula, several trees can be found.	F	Each formula has one only tree
The formula $p \vee q \wedge r$ is a positive clause	F	It's not clause
A formula is in Skolem form if it is in prenex form.	T 0.5	
The universal quantifier is one of the components of formula in Skolem form.	T 0.5	

Exercise #2 (5 marks):

1. Formalize the following sentences in propositional logic:
   a. If I do not pass, then I did not study. **(0.5)**
      p: pass, s: study
      $\neg p \Rightarrow \neg s$
   b. Amine is Algerian or if Amine isn't Algerian then Salim is Tunisian. **(0.5)**
      a: Amin is Algerian, t: Salim is Tunisian
      $a \vee (\neg a \Rightarrow s)$
   c. Amine comes to the exam if and only if Brahim comes and Salim doesn't come. **(0.5)**
      a: Amin, b: Brahim, s: Salim
      $a \Leftrightarrow (b \wedge \neg s)$

2. Formalize the following sentences in predicate logic:
   a. There exists a smart student. **(0.5)**
      $\exists x\, (Student(x) \wedge Smart(x))$
   b. Every student takes at least one course. **(0.5)**
      $\forall x.(Student(x) \Rightarrow \exists y\, (Course(y) \wedge Takes(x, y)))$.
   c. There is a printer which is not used by any student. **(0.5)**
      $\exists x.(Printer(x) \wedge \forall y.(Student(y) \Rightarrow \neg Uses(y, x)))$

3. Calculate the negation of the following formulas 1.c and 2.c.

1.c: $\neg (a \Leftrightarrow b \wedge \neg s) \equiv \neg ((a \Rightarrow b \wedge \neg s) \wedge (b \wedge \neg s \Rightarrow a))$ **(1pt)**

2.c : $\neg (\exists x.(Printer(x) \wedge \forall y.(Student(y) \Rightarrow \neg Uses(y, x))))$

$\equiv \forall x\, (\neg Printer(x) \vee \exists y\, (Student(y) \wedge Uses(y,x)))$ **(1pt)**

Exercise #3 (9 marks):

Let the following formulas:

F1: $p \Rightarrow q \wedge r$ F2: $(p \Rightarrow q) \Rightarrow r$ F3: $p \wedge q \Rightarrow r$ F4: $(p \wedge q) \Rightarrow (q \Rightarrow r)$

F5: $p \Rightarrow q \Rightarrow r$

1. Use the truth table method to verify whether F1, F2 and F3 are valid, satisfiable or unsatisfiable.

p	q	r	F1 (0.5)	F2 (0.5)	F3 (0.5)	F1⇒ F2
0	0	0	1	0	1	0
0	0	1	1	1	1	1
0	1	0	1	0	1	0
0	1	1	1	1	1	1
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	0	0	0	1
1	1	1	1	1	1	1

F1 is satisfiable, it isn't valid, and it isn't unsatisfiable. (0.25)

F2 is satisfiable, it isn't valid, and it isn't unsatisfiable. (0.25)

F3 is satisfiable, it isn't valid, and it isn't unsatisfiable. (0.25)

2. Create the parsing tree of F1∧ F4 and show its set of sub-formulas.

   Tree: (0.5)

   Sub-formulas: {p,q,r, $q \wedge r$, $p \Rightarrow q \wedge r$, $p \wedge q$, $q \Rightarrow r$, $(p \wedge q) \Rightarrow (q \Rightarrow r)$, F1∧ F4 }(0.5)

3. Verify whether the following logical consequence is correct F1 |= F2.

   Based on the truth table (column 7), F1⇒ F2 isn't tautology then the consequence F1 |= F2 is incorrect. (0.75)

4. Calculate (F2)[p/r⇒ q] (0.5)

   (F2)[p/r⇒ q] = $((p \Rightarrow q) \Rightarrow r)$[p/r⇒ q]= $((r \Rightarrow q) \Rightarrow q) \Rightarrow r$

5. Check that F3 ⇒ F4 is tautology without truth table. (1.5)

   $(p \wedge q \Rightarrow r) \Rightarrow (p \wedge q) \Rightarrow (q \Rightarrow r)$
   $(\neg p \vee \neg q \vee r) \Rightarrow (p \wedge q) \Rightarrow (q \Rightarrow r)$
   $(\neg p \vee \neg q \vee r) \Rightarrow (\neg p \vee \neg q) \vee (q \Rightarrow r)$
   $(\neg p \vee \neg q \vee r) \Rightarrow (\neg p \vee \neg q) \vee (\neg q \vee r)$
   $(\neg p \vee \neg q \vee r) \Rightarrow (\neg p \vee \neg q \vee \neg q \vee r)$
   $(\neg p \vee \neg q \vee r) \Rightarrow (\neg p \vee \neg q \vee \vee r)$
   We have p⇒ p is True (or $(\neg p \vee p = 1)$

   Therefore F3 ⇒ F4 is tautology.

6. Convert the formula $(\neg p \Rightarrow q) \Rightarrow (q \Rightarrow \neg r)$ into (CNF) using equivalent transformation. (1,5)

$(p \vee q) \Rightarrow (\neg q \vee \neg r)$
$\neg (p \vee q) \vee (\neg q \vee \neg r)$
$(\neg p \wedge \neg q) \vee (\neg q \vee \neg r)$
$(\neg p \vee \neg q \vee \neg r) \wedge (\neg q \vee \neg q \vee \neg r)$
$(\neg p \vee \neg q \vee \neg r) \wedge (\neg q \vee \neg r)$

7. Prove that S is unsatisfiable by using refutation resolution:

   S = {r, ¬q ⇒ ¬r, s ∨ ¬r, p ∨ ¬q ∨ ¬s, ¬p ∨ ¬q ∨ ¬s}. (1,5)

1. r,
2. ¬q ⇒ ¬r = q ∨ ¬r
3. s ∨ ¬r
4. p ∨ ¬q ∨ ¬s
5. ¬p ∨ ¬q ∨ ¬s
6. q Res(1,2)
7. s Res(1,3)
8. p ∨ ¬s Res(4,6)
9. p Res(7,8)
10. ¬p ∨ ¬s Res(5,6)
11. ¬p Res(7,10)
12. □Res(9,11)

<u>Exercise #4 (3.5 marks):</u>

Let the following formula: F1: ∀x ∃y P (x, f (y)) ∧ ∀y (¬Q(x, y) ⇒ R(x))

1. Create the parsing tree of F1 and show its set of sub-formulas.

Tree: (0.5)

Sub-formulas: { P (x, f (y)) , ∃y P (x, f (y)) , ∀x ∃y P (x, f (y)) , Q(x, y) , (¬Q(x, y) , R(x), ¬Q(x, y) ⇒ R(x), ∀y (¬Q(x, y) ⇒ R(x)), F1}(0.5)

2. Present different components of F1. (0.5)

   Quantifiers: ∀, ∃
   Predicates: P, Q, R
   Function: f
   Connectors: ∧, ¬, ⇒

3. Find free variables in F1. (0.5)

The occurrences of x in Q(x,y) and R(x) remain **free**.

4. Is F1 is closed? And why? (0.5)

   No, there exist a free variable.

5. Compute $\neg F1$

   ¬ (∀x ∃y P (x, f (y)) ∧ ∃y (¬Q(x, y) ⇒ R(x)))

   ≡ ¬ (∀x ∃y P (x, f (y)) ∧ ∃y (Q(x, y) ∨ R(x))) (0.5)

   ∃x ∀y ¬P (x, f (y)) ∨ ∃y (¬Q(x, y) ∧ ¬ R(x)) (0.5)