# Predicate Logic – Syntax (Part 1)

**Introduction :**

Example 1:

- Ibrahim is a student➔P1
- Zahra is a student➔P2
- Imad is a student➔P3
- Aya is a student➔P4
- …. 100 students ????
- The predicate is a generalization of the propositional variable, P1, P2, …., P100➔P(x): "x is a student"

**Example 2:**

- 5 > 3 It is a statement, we can model it as a proposition, so its truth value is true.
- 2 > 3 It is a statement, we can model it as a proposition, so its truth value is false.
- x>3 This is a statement, we cannot model it as a proposition. The truth value depends on the value of x.

**Example 3:**

Consider the following sentences:

- Person x is father of y
- Person z is father of x
- So, person z is grandfather of y

- It is very difficult to translate these sentences into propositional logic.
- Another sentence: All students are intelligent

- In propositional logic, we only use propositions that are facts.
- Predicate logic (first-order logic) (LP1) is considered an extension of propositional logic that gives the possibility of using other notions (terms, functions, relations, etc.).

## Predicate Logic Syntax

The alphabet of the language of predicate logic is composed of:

- Logical connectors:$\neg$, $\wedge$, $\vee, \Rightarrow, \Leftrightarrow$
- variables: , $\ldots x, y, z$
- Constants: Amine, A, B, …
- functions: f, $g$,
- Predicates: $P, P_1, P_2, Q,$
- Quantifiers:$\forall, \exists$
- Auxiliary symbols: (, ).

- **Constant:** is a element in domain $D$.
  - Example: If $D$ is ,$\mathbb{N}$

    1, 4, 500 are examples of constants in domain $D$
- **Function:** is an application.$D^n to\ D$
  - Examples: If $D$is $\mathbb{N}$

    $f(x) = x + 1\ and\ g(x, y) = x + y$ are functions.
- <u>**The terms**</u> are defined as:
  1. Variables and constants (domain element $D$) are terms.
  2. If $f$ is a function with *n* arguments of LP1 and $t_1, t_2, \ldots, t_n$ are terms then $f(t_1, t_2, \ldots, t_n)$ is a term of LP1.
  3. All terms are generated by rules (1) and (2).

  **Examples:** $x, Af(x), g(y, f(x))$

- **The predicate:** a generalization of the propositional variable, P1, P2, …., P100➔P(x): "x is a student"
- A predicate is a statement where its truth value {0, 1} depends on one or more variables (elements of a domain).
- A defined application from a domain $D^n$ to the set $\{0,1\}$.
- Every predicate has a fixed number of arguments: arity.

**Examples:**

- Amine is a student Student(Amine)

  (constant) (predicate) arity:1

- $x$ is even number Even($x$)

  (variable) (predicate) arity:1

- $x$ is greater than $y$ Greater($x, y$)

  (variable) (predicate) (variable) arity:2

**Quantifiers**

- Quantifiers are mathematical symbols used to simplify the writing of mathematical propositions.
- There are two quantifiers:
  - The universal quantifier (∀) (For all)
  - The existential quantifier (∃) (There exist)

**Universal quantifier:**

  - $\forall < variables >< statement >$
  - Reads: “for all…”, “whatever…”, “whoever…”, “all”, etc,

Examples:

  - “All students are smart” is modeled by: $\forall x\ (Etudiant(x) \Rightarrow\ Intelligent(x))$
  - “All A(x) is B(x)” is modeled by: $\forall x\ (A(x) \Rightarrow B(x))$

- $\forall x\ P$ is equivalent to the conjunction of all instantiations of $P$ :

$$\forall x P(x)\ \Leftrightarrow\ P(n_1)\ \wedge\ P(n_2)\ \wedge \cdots \wedge\ P(n_k)$$

**Existential quantifier:**

- $\exists < variables >< \text{é}noncé >$
- reads: “there exists…”, “there exists at least one…”, “some…”

**Examples:**

- "There are students who are intelligent" is modeled by: $\exists x\ (Etudiant(x) \wedge\ Intelligent(x))$
- “Some A(x) are B(x)” is modeled by: $\exists x\ (A(x) \wedge B(x))$

- $\exists x\ P$ is equivalent to the disjunction of all instantiations of $P$:

$$\exists x P(x) \;\Leftrightarrow\; P(n_1) \;\vee\; P(n_2) \vee \cdot\cdot\cdot \vee P(n_k)$$

**Note:**

- The main connector to be used with ∃ is the conjunction ∧.
- Use implication ⇒ as main connector with ∀.

**Properties of quantifiers:**

- $\forall x \forall y$ *is equivalent to* $\forall y \forall x$
- $\exists x \exists y$ *is equivalent to* $\exists y \exists x$
- ***Attention :*** $\exists x \forall y$ *is not equivalent to* $\forall y \exists x$
- $\exists y \forall x\ Eat(x, y)$ *means "There is one thing that everyone eats it"*
- $\forall x \exists y\ Eat(x, y)$ *means "Everyone eats something" (for every person,* there is something he eats)
- Negation of quantifiers: The two quantifiers are linked through negation.
    - $\neg \forall x\ P(x) \equiv \exists x \neg P(x)$
    - $\neg \exists x\ P(x) \equiv \forall x \neg P(x)$
- Example:
    - $\forall x\ Eat(x, Glace)$ *is equivalent to* $\neg \exists x \neg Mange(x, Glace)$
    - $\exists x\ Eat(x, Glace)$ *is equivalent to* $\neg \forall x \neg Mange(x, Glace)$

**Exercise:**

- x is a man -------------> model as a predicate H(x)
- x is a person -------------> P(x)
- x drinks coffee -------------> Drink(x, Coffee)
- x is a woman -------------> F(x)
- x eats y -------------> Eat(x,y)
- Amine is a man -------------> H(Amine), Amine is a constant
- All men are persons ------------->∀x (H(x)⇒P(x))
- Some men don't drink coffee ------------->∃x (H(x)∧¬B(x, Coffee))

- No woman is a man:------------->$\forall x\ (F(x) \Rightarrow \neg H(x))$
- All women drink coffee: ------------->$\forall x\ (F(x) \Rightarrow B(x, Coffee))$