Propositional Logic – Resolution (Part 2)

A. Resolution methods:

1. Deduction resolution: $C \vdash P$

- To demonstrate that P is deducible from a set of clauses C, we use resolution proof.
- From the initial clauses C, we apply the resolution rule to calculate the resolvent of two clauses. We apply the same principle for the resolving clauses until we obtain P.

Example 1: Demonstrate that: $\{q \vee \neg r; \neg q \vee t; \neg t\} \vdash \neg r$

1) q ∨ ¬r
2) $\neg q \vee t$
3) $\neg t$
4) $\neg r \vee t$ Res(1,2)
5) $\neg r$ Res(3,4)

2. Refutation resolution: $C \cup \{\neg P\}$

- From $C \cup \{\neg P\}$, we apply a sequence of resolution rules to compute resolving clauses in order to obtain an empty clause □, if possible.
- A set of clauses admitting of refutation $C \cup \{\neg P\}$ is insatisfiable.

Algorithm:

- Let $C' = C \cup \{\neg P\}$
- While (□)$\notin C'$) and (there exist new possible choices)
  - Find two clauses to apply the resolution rule
  - Add the resolving clause to the clause list.
- If □ $\in C'$ then is insatisfiable C'

Otherwise is satisfiable C'

Example 1: Demonstrate that: $\{\boldsymbol{q} \vee \neg \boldsymbol{r}; \neg \boldsymbol{q} \vee \boldsymbol{t}; \neg \boldsymbol{t}\} \vdash \neg \boldsymbol{r}$

- $C' = C \cup \{\neg\neg r\} = \{q \vee \neg r; \neg q \vee t; \neg t; r\}$

1) q ∨ ¬r
2) $\neg q \vee t$
3) $\neg t$

4) r

5) $\neg r \vee t$ (1,2)

6) $\neg\, r$ (3,5)

7) □ (4,6)

Note:

- We can use a clause multiple times.
- We don't require using all clauses to deduce the empty clause.

B. <u>Selection strategies</u>

1. Positive resolution:

- To apply the resolution rule, one of the chosen clauses must be a <u>positive clause</u> (with no negative lateral).
- This strategy is complete for refutation.

Example 1: Demonstrate that: $\{q \vee \neg r; \neg q \vee t; \neg t\} \vdash \neg r$ using the positive resolution strategy.

- C': $\{q \vee \neg r; \neg q \vee t; \neg t; r\}$

1) q ∨ ¬r

2) $\neg q \vee t$

3) $\neg t$

4) r

5) q (1,4)

6) t (2,5)

7) □ (3,6)

2. Negative resolution:

- To apply the resolution rule, one of the chosen clauses must be a <u>negative clause</u>, with no positive lateral.
- This strategy is complete for refutation.

Example: Demonstrate that: $\{q \vee \neg r; \neg q \vee t; \neg t\} \vdash \neg r$ using the negative resolution strategy.

- C': $\{q \vee \neg r; \neg q \vee t; \neg t; r\}$

1) q ∨ ¬r

2) $\neg q \vee t$

3) $\neg t$

4) r

5) $\neg q$(2,3)

6) $\neg r$ (1,5)

7) □ (4,6)

3. Linear resolution for Horn clauses:

- The principle of linear resolution: reduce the clauses at each step by eliminating one of the two clauses which is used to calculate the resolvent.
- Horn Clause:
  - Any clause that has at most one positive literal.
  - Examples: $p \vee \neg q, p \vee \neg q \vee \neg r, \neg p$
- Positive unit clause: it contains one positive literal and no negative literal.
- The goal of the linear resolution algorithm for Horn clauses is to demonstrate that a set of clauses is insatisfiable.
- This algorithm is complete for refutation.

Algorithm

- Let there be a set of clauses C'.
- While (□$\notin C'$) and (there are new possible choices)
  - Select two clauses: p is a positive unit clause and h is a clause containing $\neg p$.
  - Calculate the resolving clause r from p and h.
  - Replace C' with $(C' - \{h\}) \cup \{r\}$
- If □ $\in C'$ then $C'$is insatisfiable.

Otherwise C' is satisfiable

Example: Demonstrate that the set C' is insatisfiable.

$$C'\colon \{q \vee \neg r; \neg q \vee t; \neg t; r\}$$

1) $\{q \vee \neg r; \neg q \vee t; \neg t; r\}$

2) $\{q; \neg q \vee t; \neg t; r\}$

3) $\{q; t; \neg t; r\}$

4) $\{q; \square; r\}$

- Then, the set of clauses C' is insatisfiable.

Exercise:

- Demonstrate using the resolution methods that:

1. $\{\neg p \vee \neg q \vee r; \neg r; p; \neg t \vee q\} \vdash \neg t$
2. $\{p \Rightarrow q; q \Rightarrow r\} \vdash \ p \Rightarrow r$