Propositional Logic - Syntax

Introduction :

- In propositional logic (also called "propositional calculus", only two types of things are used, statements and connectives.
- Every statement in propositional logic consists of **propositional variables** combined via **logical connectives**.
- The formalization capacity of propositional logic is very limited compared to predicate logic.
- **The syntax** defines how to write the form of the language.

The Proposition:

- A proposition (or assertion) is a statement that is, by itself, either true or false, not both at the same time.
- A statement is a sentence or expression to which we can attribute a meaning.
- Example :
    - Algiers is the capital of Algeria
    - 5-2=3
    - $1+2\geq 7$
    - I am a student

The following types of sentences are excluded **(cannot be true or false)**:

- **Questions**: What is your family name?
- **Commands:** Restart your laptop!
- **Exclamatory sentence:** Oh, That is excellent!
- **We can discuss it:** The match will take place next week
- **Non-assertive statement:** 3+7

Propositional variable:

- A propositional variable is represented as lower-case letters (p, q, r, etc.).
- If we need more, we can use subscripts: p_1, p_2, etc.
- Each variable can take one of two values: true or false.

Logical connectives

- To formalize some sentences, we need to use logical connectives.
- Logical connectives are operators that allow, by relating two propositions in order to create a new proposition.

Logical connectives	**Definition**	**Natural language**	**Example**
Negation	The negation of a proposition noted $\neg P$ (not P).		"He didn't send a text message" is the negation of the proposition, we note it by $\neg P$ P: He sent a text message
Conjunction	The conjunction of two propositions P, Q is denoted by $P \wedge Q$.	"P and Q", "Both P and Q." "P, but Q", "Not only P but Q",	"x <5 and x>0". If P is the proposition "x <5" and Q is the proposition "x>0", then the proposition is: $P \wedge Q$
Disjunction	The disjunction of two propositions P, Q is denoted by $P \vee Q$.	"P or Q", "P unless Q", "either P or Q".	"x <5 or x>0». If P is the proposition "x <5" and Q is the proposition "x>0", The proposition: $P \vee Q$
Implication	The proposition "P⇒Q" reads "P implies Q". It corresponds to the proposition not P or Q.	If P then Q, P is a sufficient condition for Q, p only when q P only if Q	<u>If the network is configured correctly </u>then the internet connection is successful. $P \Rightarrow Q$
Biconditional	The proposition "P⇔Q"	*"P equals Q»* "P if and only if Q" "*P* is a necessary and sufficient condition for Q»	*<u>x is a positive number </u>if and only if x>0* P: "x is a positive number" Q: "*x>0*" *Translation:* P⇔Q

Operator Precedence

- Let the following statement: $P \wedge Q \vee R$
  - There is an ambiguity in the reading of this statement.
  - There are two ways to read this statement: $(P \wedge Q) \vee R$ *o*u $P \wedge (Q \vee R)$.

- To avoid this problem, either use parentheses or accept the operator precedence.

- Operator precedence is presented in the following order :$\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$
- Negation () has the highest priority over other connectors.$\neg$
- All operators are right-associative.

Examples: $\neg P \vee Q$, ,$P \Rightarrow Q \vee R$ $P \vee Q \wedge R$ $P \vee \neg Q \wedge R P \Rightarrow Q \Rightarrow R$,

Exercise 1: translate the following sentences into propositional logic.

1. It's not raining
2. It's raining and not snowing
3. If it rains then I take an umbrella.
4. I take an umbrella only if it rains.
5. It rains or it snows, but not at the same time
6. It's not raining, so I'm not taking the umbrella