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جامعة المسيلة
كلية الرياضيات و الإعلام الآلي
قسم الإعلام الآلي

$Exam : Data Structures and Algorithms 3 (DSA 3)$

Duration: 2h - Documents: not allowed **Academic year : 2025 / 2026**

Date : 17/01/2026 (10 : 30 – 12 : 00) **Corrected** **Level : L2** **Semester : 5**

First Name	Last Name	Group
.....	.....	.....

Exercise No.	1	2	3	4	5	6	Total
Marks Obtained	1.5	5	5	4	4.5	2	20

Exercise 1 : (1.5 Points / 10 Minutes)

Write the values of the **int** variables when the following program ends.

```
#include <stdio.h>
void main() {
   int a[] = {4, 5, 8, 13, 20};
   int *p, *q;
   int i, j, k, l, m, n;
   q = &a[4];
   p = q - 4;
   i = *q;
   j = *p++;
   k = *--q;
   l = p[1];
   m = *(q-2);
   n = q - p;
}
```

Answer (0.25 × 6 = 1.5 pts)

i = ...20.....

j = ...4.....

k = ...13.....

l = ...8.....

m = ...5.....

n = ...2.....

Exercise 2 : (5 Points / 20 Minutes)

Q1) Prove that $T(n) = n^2 + \log(n^2)$ is $O(n^2)$ using the formal definition of the Big-Oh notation. [**Hint**: Find a constant c and threshold n_0 such that $T(n) \leq cn^2$ for $n \geq n_0$.]

However, we can show that $\boldsymbol{T(n) \in \mathcal{O}(n^2)}$. (1 pt)

We have : $\boldsymbol{T(n) = n^2 + \log(n^2)}$

$\boldsymbol{\leq n^2 + n^2}$ for all $\boldsymbol{n \geq 1}$

$\boldsymbol{= 2n^2}$

Choosing $\boldsymbol{c = 2}$ and $\boldsymbol{n_0 = 1}$, for all $\boldsymbol{n \geq n_0}$, we have : $\boldsymbol{T(n) = n^2 + \log(n^2) \leq c.n^2}$

so that $\boldsymbol{T(n) \in \mathcal{O}(n^2)}$.

Dr. A. DABBA

Q2) Calculate the complexity of the following sources fragments; briefly justify your answer:

Pieces of code	Complexity

`int sum = 0;`
`for(int i = n*n*n*n; i >= 1; i /= 4)`
`  sum++;`
(0.25 + 0.25 = 0.5pts)
	• **i** starts at n^4
	• Each step: **i = i / 4**
	• Number of iterations ≈ $log_4(n^4)$
	$log(n^4) = 4\ log\ n \Rightarrow O(log\ n)$

`for(int i = 1; i < size; i++){`
`  current = arr[i];`
`  for(j = i; j>0 && arr[j-1]>current]; j--)`
`  arr[j] = arr[j-1];`
`  arr[j] = current;`
`}` **(0.25 + 0.25 = 0.5pts)**
	Total operations:
	$1 + 2 + \cdots + (n-1) = \sum_{i=1}^{n-1} i$
	$\frac{n(n-1)}{2} \Rightarrow O(n^2)$

`int sum = 0;`
`for (int i = 0; i < n/2; i++){`
`  for (int j = 0; j < n; j++)`
`  sum++;`
`  for (int j = 1; j < n * n; j++)`
`  for(int k = 1; k < 2026; k++)`
`  sum++;`
`}` **(0.25 + 0.25 = 0.5pts)**
	• Outer loop runs **n/2** → $O(n)$
	• First inner loop runs **n** → $O(n)$
	• Second inner loop runs n^2 → $O(1 \times n^2) = O(n^2)$ **because**
	➢ j loop runs **n²**
	➢ k loop runs **2025 times** (constant)
	$Max\left(O(n), O(n^2)\right) \times O(n) = O(n^3)$

`int func(int n){`
`  for (int i = 0; i < n; i ++)`
`  // some operations`
`  if (n <= 0)`
`  return 1;`
`  else`
`  return 1 + func(n-1) + func(n-1);`
`}` **(0.5 + 0.5 = 1pt)**
	Recurrence relation
	$T(n) = \begin{cases} O(1) & if\ n = 0 \\ 2\,T(n-1) + O(n) & otherwise \end{cases}$
	This is a **binary recursion**.
	• The $2\,T(n-1)$ term dominates
	• The recursion tree **doubles at each level**
	• Depth of recursion = **n**
	Total number of calls: 2^n
	$\Rightarrow O(2^n)$

`int count = 0;`
`for (int i = n; i > 0; i /= 2)`
`  for (int j = 0; j < i; j++)`
`  count++;`
(0.25 + 0.25 = 0.5pts)
	Total operations:
	$n + \frac{n}{2} + \frac{n}{4} + \cdots + \frac{n}{2^k} = \sum_{k=0}^{log\ n} \frac{n}{2^k}$
	$\sum_{k=0}^{log\ n} \frac{n}{2^k} \leq 2n \Rightarrow O(n)$

Q3) You must find **1000** most expensive items from an unsorted price list containing $\mathbf{10^7}$ different items. Two schemes of solution are as follows.

- **Scheme A**: repeat **1000 times** the sequential search (with the linear complexity $\boldsymbol{O(n)}$).
- **Scheme B**: convert the list into an array (the same complexity $\boldsymbol{O(n)}$ as for search), then sort the array (complexity $\boldsymbol{O(n \log n)}$) and fetch **1000** top items.

Which scheme would you prefer assuming that searching for **100 items** in the unsorted list takes **0.1 millisecond (ms)** and sorting of **100 items** also takes **0.1 ms**? **Time for fetching data should not be taken into account**. **(0.25 + 0.25 + 0.5 = 1pt)**

The scaling factors for the linear search and the sort are $\frac{0.1}{100} = 0.001$ and $\frac{0.1}{100 \log 100} = \frac{1}{200} = 0.0005$, respectively (you may use any convenient base of the logarithm for computing the running time).

Then the overall time for the **scheme A** is

$$\boldsymbol{TA} = 0.001 \times 10^7 \times 10^3 = 10^7 ms = \mathbf{10^4}\ \boldsymbol{sec}\ .$$ **(0.25 pts)**

The overall time for **scheme B** consists of the time to convert the list into an array:

$$\boldsymbol{TB1} = 0.001 \times 10^7 = 10^4 ms = \mathbf{10}\ \boldsymbol{sec}$$

and the time for sorting the array:

$$\boldsymbol{TB2} = 0.0005 \times 10^7 \log(10^7) = 35 \times 10^3 ms = \mathbf{35}\ \boldsymbol{sec}$$

So, $\boldsymbol{TB} = TB1 + TB2 = 10 + 35 = \mathbf{45}\ \boldsymbol{sec}$ **(0.5 pts)**

Therefore, we have $\boldsymbol{TB < TA}$ the scheme of choice is B. **(0.25 pts)**

<u>Exercise 3 :</u> (5 Points / 20 Minutes)

The stooge sort is a recursive sorting algorithm that can be written quite simply:

```
void stooge_sort(int Tab[], int start, int end) {
    if (Tab[start] > Tab[end]) {
        swap(Tab[start], Tab[end]);
    }
    if (end - start + 1 > 2) {
        int t = (end - start + 1) / 3;
        stooge_sort(Tab, start, end - t);
        stooge_sort(Tab, start + t, end);
        stooge_sort(Tab, start, end - t);
    }
}
```

Q1) Run the algorithm for `stooge_sort([8, 9, 2, 7], 0, 3)`. During the execution, you must rewrite the contents of the array **`Tab`** each time values are swapped. **(2 pts)**

For each new recursive call, you must indicate the values of the **`start`** and **`end`** indices.

Call `stooge_sort(Tab, start, end)`	**Tab are swapped**
`stooge_sort([8, 9, 2, 7], 0, 3)`	**`[7, 9, 2, 8]`**
`stooge_sort([7, 9, 2, 8], 0, 2)`	`[2, 9, 7, 8]`
`stooge_sort([2, 9, 7, 8], 0, 1)`	
`stooge_sort([2, 9, 7, 8], 1, 2)`	`[2, 7, 9, 8]`
`stooge_sort([2, 7, 9, 8], 0, 1)`	
`stooge_sort([2, 7, 9, 8], 1, 3)`	
`stooge_sort([2, 7, 9, 8], 1, 2)`	
`stooge_sort([2, 7, 9, 8], 2, 3)`	`[2, 7, 8, 9]`
`stooge_sort([2, 7, 8, 9], 1, 2)`	
`stooge_sort([2, 7, 8, 9], 0, 2)`	
`stooge_sort([2, 7, 8, 9], 0, 1)`	
`stooge_sort([2, 7, 8, 9], 1, 2)`	
`stooge_sort([2, 7, 8, 9], 0, 1)`	

Q2) Give the recurrence equation for the number of recursive calls performed by **`stooge_sort(Tab, 0, n - 1)`**, where **`n`** is the number of elements in **`Tab`**.

$$T(n) = \begin{cases} O(1) & \text{if } end - strat + 1 \leq 2 \\ 3\,T\left(\frac{2n}{3}\right) + O(1) & \text{otherwise} \end{cases}$$ **(1 pt)**

Q3) Deduce the asymptotic complexity of sorting from the recurrence equation.

For this recurrence, we have $a = 3, b = 3/2$, and $f(n) = O(1) \Rightarrow c = 0$ **(1 pt)**

Since $a > b^c$, $[3 > (3/2)^0]$

As per ***case* 1** of the master theorem, the solution is : $T(n) = O\left(n^{log_{3/2}(3)}\right) = O\left(n^{2.7095}\right)$

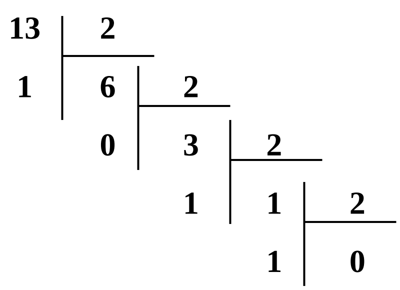
Q4) What should we think about the complexity of this sorting method compared to Bubble sort and Quick sort?

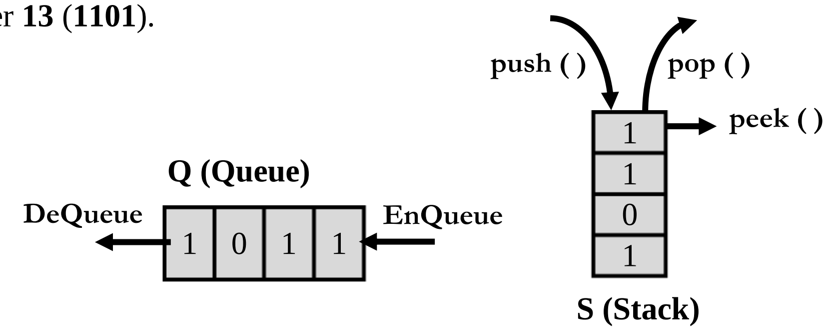
The asymptotic complexity of the stooge sort is **worse than** that of Bubble sort ($O(n^2)$) also worse than that of Quick sort in the worst case (also ($O(n^2)$)). **(1 pt)**

Therefore, Stooge Sort is a **particularly inefficient sorting algorithm**.

Exercise 4 : (4 Points / 25 Minutes)

In the example below, we have a **queue Q** and a **stack S**, each containing the binary representation of the integer number **13** (**1101**).





Q1) Provide the declaration for this queue and this stack. **(0.25 + 0.25 + 0.25 = 0.75 pts)**

Node	Queue	Stack

`struct Node {`
`bool val;`
`struct Node* next;`
`};`
	`typedef struct {`
	`struct Node *first;`
	`struct Node *last;`
	`int size;`
	`} QUEUE;`
		`typedef struct {`
		`struct Node *top;`
		`int size;`
		`} STACK;`

In all questions, you can use all functions seen in course without needing to redefine them.

Q2) Write the function **`binary(x, Q)`** which receives a decimal number **`x`** and creates a queue **Q** containing its binary representation. **(1 pt)**

```
void binary(int x, QUEUE *Q) {
    if (x == 0) {                // Handle special case x = 0
        EnQueue(Q, 0);
        return;
    }
    while (x > 0) {
        EnQueue(Q, x % 2);
        x = x / 2;
    }
}
```

Q3) Write a **`Stack_Reverse(S)`** that destructively returns a stack with the same elements as its input stack but in reverse order. **(0.75 pts)**

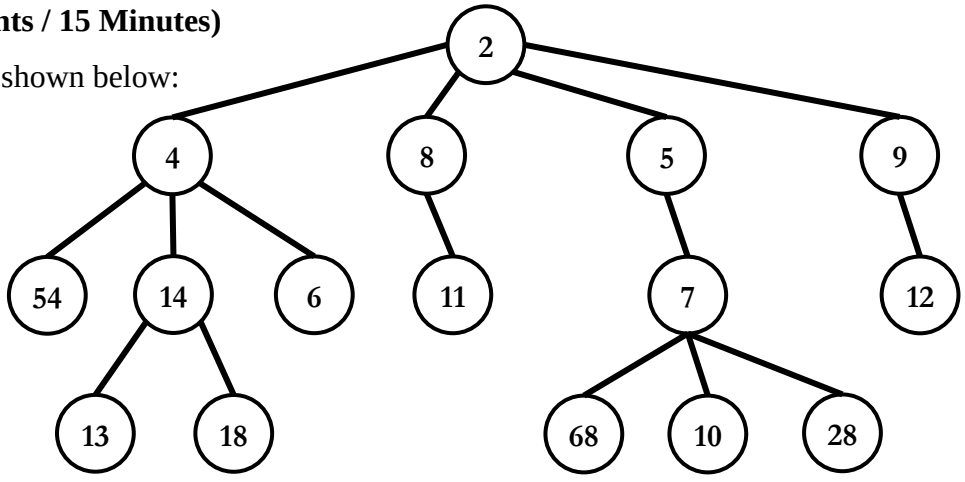
```
STACK Stack_Reverse (STACK S){
    STACK Reverse = init_Stack();                    // NULL
    while (!isEmpty(S)){
        push(Reverse, pop(S));
    }
    return Reverse;
}
```

Q4) Write the function **`egual(Q, S)`** that makes it possible to verify whether the queue **Q** and the stack **S** contain binary representations of the same integer number. **(1.5 pts)**

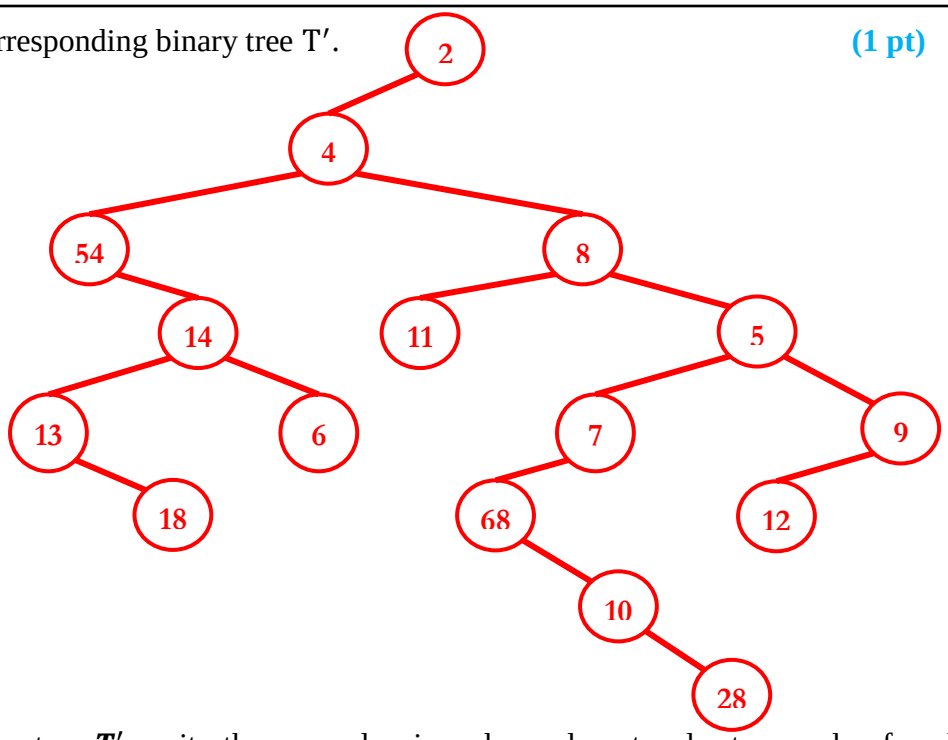
```
bool egual(QUEUE Q, STACK S) {
    if (Q.size != S.size)            // Sizes must be equal
        return false;
    STACK temp = Stack_Reverse(S);
    while (Q.size > 0 && S.size > 0) {
        if (DeQueue(Q) != pop(S))    // Compare each bit
            return false;
    }
    return true;

}
```

<u>Exercise 5 :</u> (4.5 Points / 15 Minutes)

For the general tree ***T*** shown below:



Q1) Find the corresponding binary tree T′. **(1 pt)**



Q2) In the binary tree ***T′***, write the pre-order, in-order and post-order traversals of each one. **(0.5 + 0.5 + 0.5 = 1.5 pts)**

Pre-order	2→ 4→ 54→ 14→ 13→ 18→ 6→ 8→ 11→ 5→ 7→ 68→10→ 28→ 9→ 12
In-order	54→ 13→ 18→ 14→ 6→ 4→ 11→ 8→ 68→10→ 28→ 7→ 5→ 12→ 9→ 2
Post-order	18→ 13→ 6→ 14→ 54→ 11→ 28→10→ 68→ 7→ 12→ 9→ 5→ 8→ 4→ 2

Q3) Is the ***T′*** a binary search tree? Why? **(0.5 pts)**

No, ***T′*** is **not a binary search tree**, because the node with value **54** is in the **left subtree of node 4. W**hich violates the BST **property.**

A node in a binary tree is defined as follows:

```
struct BTNode {
    int data;
    BTNode *left;
    BTNode *right;
};
typedef BTNode *BTREE;
```

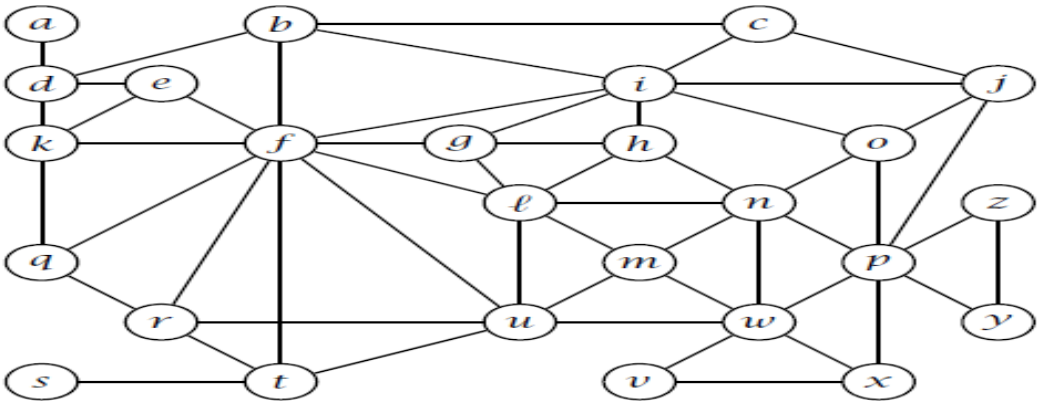
Q4) Write a **`Height(BTree)`** function to find a height of any node in the binary tree.

```
int Height(BTREE BTree) {
```
(1.5 pts)

```
    if (BTree == NULL)
       return -1;
    else
       if ((BTree ->left== NULL)&&(BTree ->right== NULL))
          return 0;
     return max(Height(BTree->left),Height(BTree->right)) + 1;
}
```

Exercise 6 : (2 Points / 10 Minutes)

Consider the following graph. Suppose we want to traverse it, starting at node ***l***.



Q1) If we traverse this using **breadth-first search (BFS)**, what is a possible order of the nodes we visit? When processing the neighbors of a vertex, process them in alphabetical order.

(1.5 pts)

l→f→g→h→m→n→u→b→e→i→k→q→r→t→w→o→p→d→c→j→s→v→x→y→z→a

Q2) Write the structure of the graph to represent it using the adjacency list. **(0.5 pts)**

```
struct GNode{
    int vertex;
    GNode* next;
};
// Structure to represent the graph
struct Graph {
    unsigned int numVertices;
    GNode** adjLists;
    bool isDirected;
};
```

Good Luck