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MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESERACH

Mohamed Boudiaf University of M'sila

Faculty of Mathematics and Informatics

Departement of Mathematics

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Field: Mathematics
Specialty: Partial Differential Equations and Applications

Variational methods for nonlinear elliptic problems

A Course with Exercises and Key Topics

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Preface

What is the main objective of this course? Anyone interested in studying ordinary and partial differential equations will undoubtedly need some tools from functional analysis, as well as familiarity with certain methods used to prove the existence and multiplicity of solutions and to determine some of their properties. Among the most important of these methods are the so-called *variational methods and critical point theory.* We try to find solutions to a given nonlinear problem by looking for critical points of a functional defined on the functional space in which we want the solution to lie. So, for a given problem, one usually proceeds as follows:
Firstly, we find the real valued functional that is naturally associated with the problem.
Secondly, we find or construct a theorem that ensures our functional has a critical point.
Finally, we prove that a critical point of our functional is a solution to the problem.

This course is primarily intended for Master's students in Partial Differential Equations (PDEs) and Functional Analysis of PDEs, as well as related specialties. It can also serve as a reference for beginner PhD students who are looking for a simplified introduction to this field. In these lectures, I have aimed to simplify the most important variational methods by selecting simple model problems that help the learner grasp the objectives smoothly, without delving into complexities that might hinder their understanding and deeper exploration. Therefore, while it may not offer new results, its value lies in the rephrasing of the lectures, the careful selection of examples, and the presentation of key problems in a way that is accessible and comfortable for the reader.

I would also like to draw the reader's attention to the fact that I do not recall any work ever being entirely accurate; thus, the discovery of an error should come as no surprise. I would be sincerely grateful to anyone who brings such issues to my attention.

MOKHTARI Abdelhak
M'sila, on: 23 April 2025

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