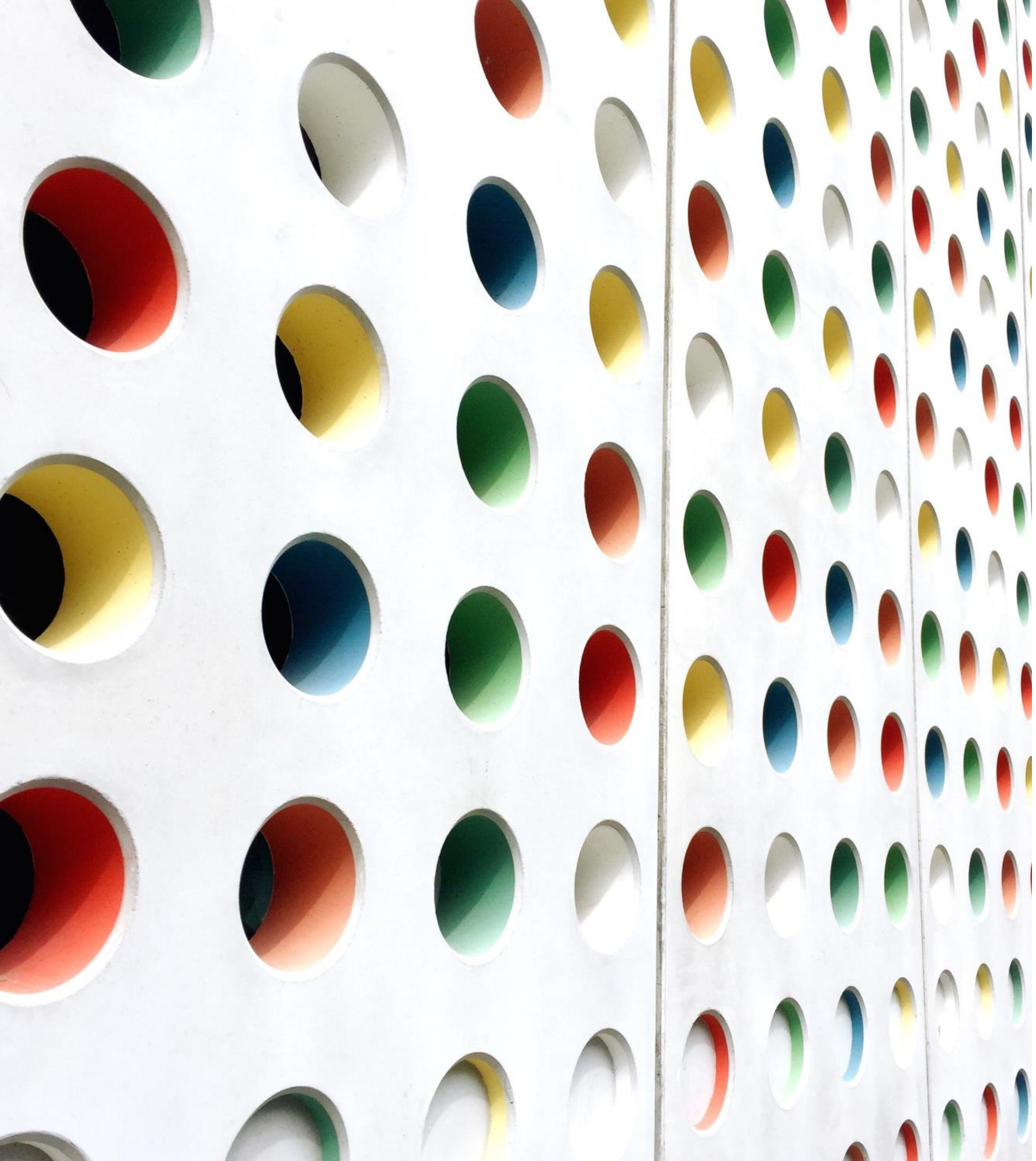


Fuzzy Logic

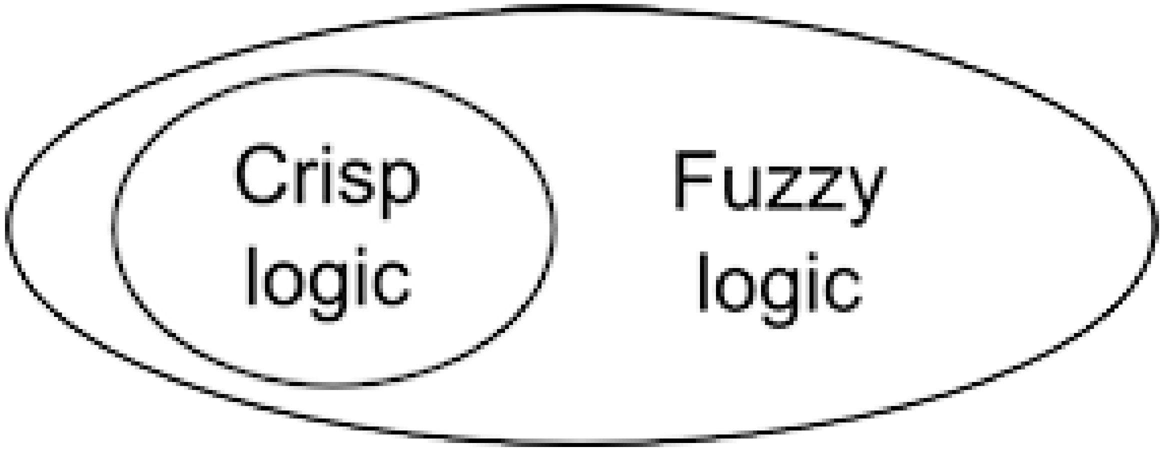
ABDELBASSET B

Introduction

Fuzzy logic is an extension of Boolean logic by Lotfi Zadeh in 1965 based on the mathematical theory of fuzzy sets, which is a generalization of the classical set theory. By introducing the notion of degree in the verification of a condition, thus enabling a condition to be in a state other than true or false, fuzzy logic provides a very valuable flexibility for reasoning, which makes it possible to take into account inaccuracies and uncertainties.

Introduction

Fuzzy logic is based on the theory of fuzzy sets, which is a generalization of the classical set theory. Saying that the theory of fuzzy sets is a generalization of the classical set theory means that the latter is a special case of fuzzy sets theory. To make a metaphor in set theory speaking, the classical set theory is a subset of the theory of fuzzy sets,



Importance

Traditional control systems (based on classical logic) make binary decisions:

- If temperature > 25°C → turn ON the fan at high speed
- Else → turn it OFF

Fuzzy logic is used:

- IF temperature is Cold → fan speed is Slow
- IF temperature is Warm → fan speed is Medium
- IF temperature is Hot → fan speed is Fast

Fuzzy Sets

- **Fuzzy sets**
- A fuzzy set allows elements to **partially belong** to the set.
- Example: in the fuzzy set *"tall people"*, a person who is 180 cm might belong with a degree of 0.6, while someone 190 cm might have 1.

- **Membership function (μ)**
- Defines how each element is mapped to a membership value between 0 and 1.
- Example:

$\mu_{tall}(x)=0.6$ means person x is tall to degree 0.6.

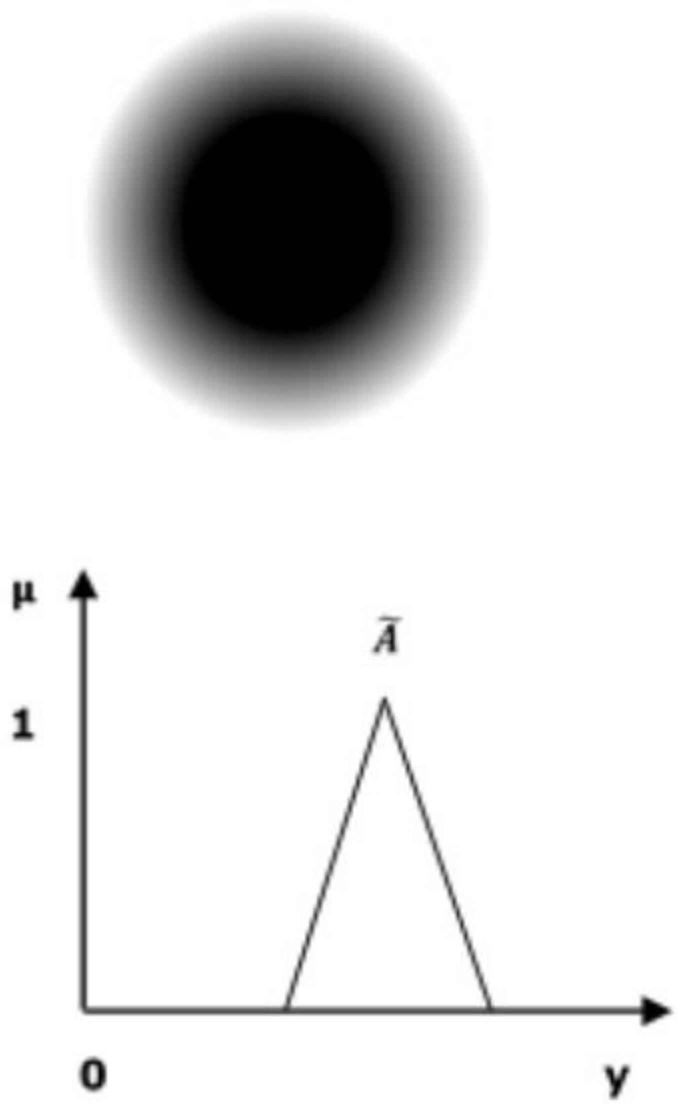
Fuzzy Sets

Definition . Let U be a set. A fuzzy subset A of U is characterized by a membership function. f : U → [0,1].

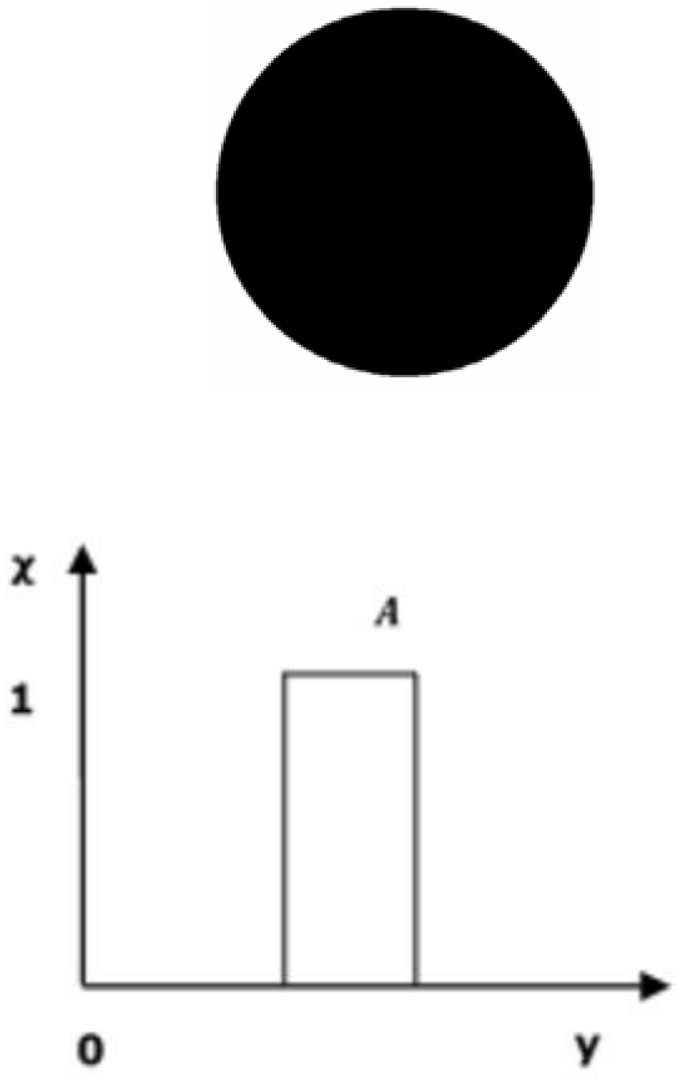
$$\forall\, x \in U\colon \mu_A(x) = [0,1]$$

- **Membership function (μ)**
- Defines how each element is mapped to a membership value between 0 and 1.
- Example:

$\mu_{tall}(x)=0.6$ means person *x* is tall to degree 0.6.



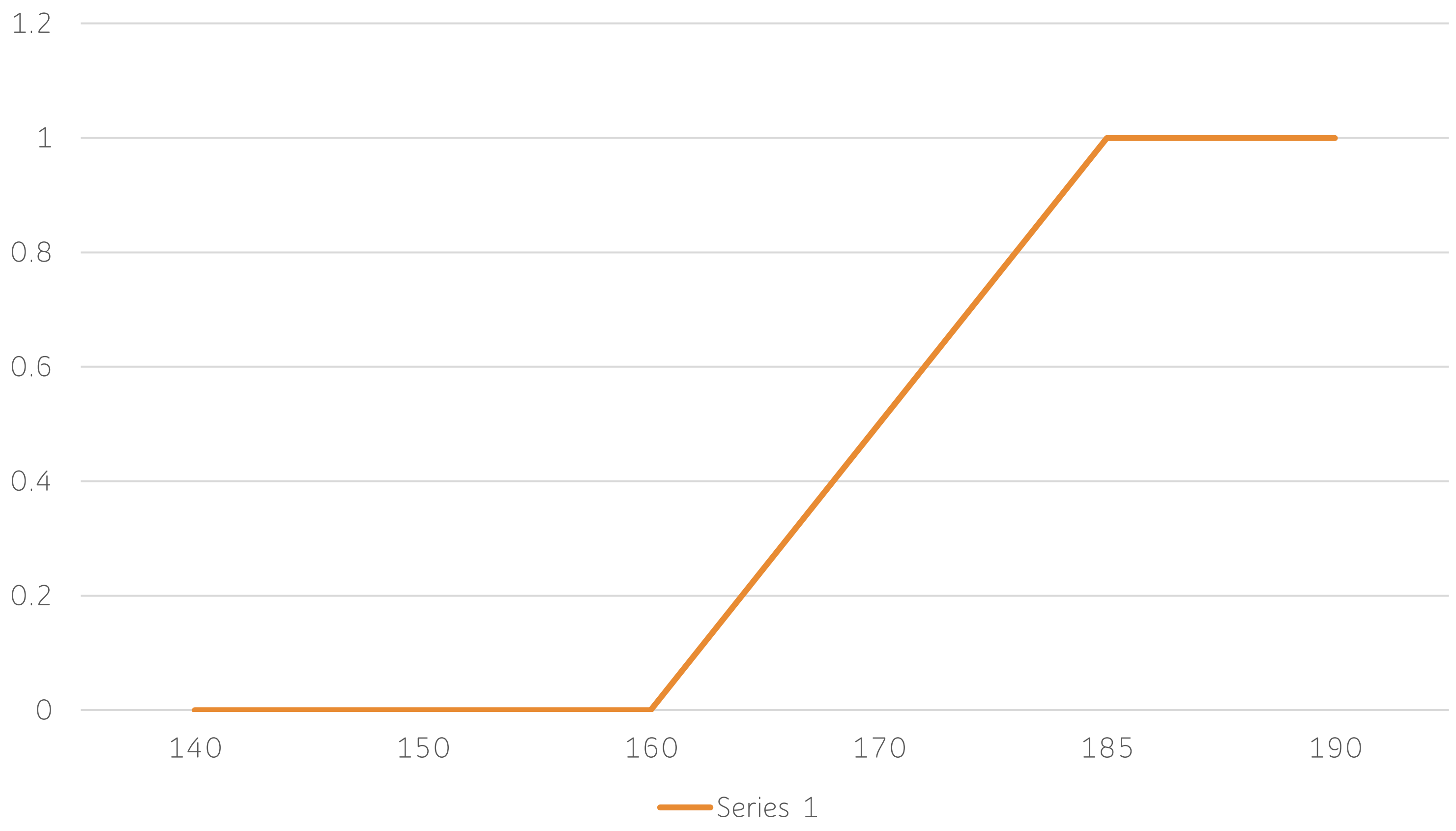
Membership Function of Fuzzy set $\tilde{A}$



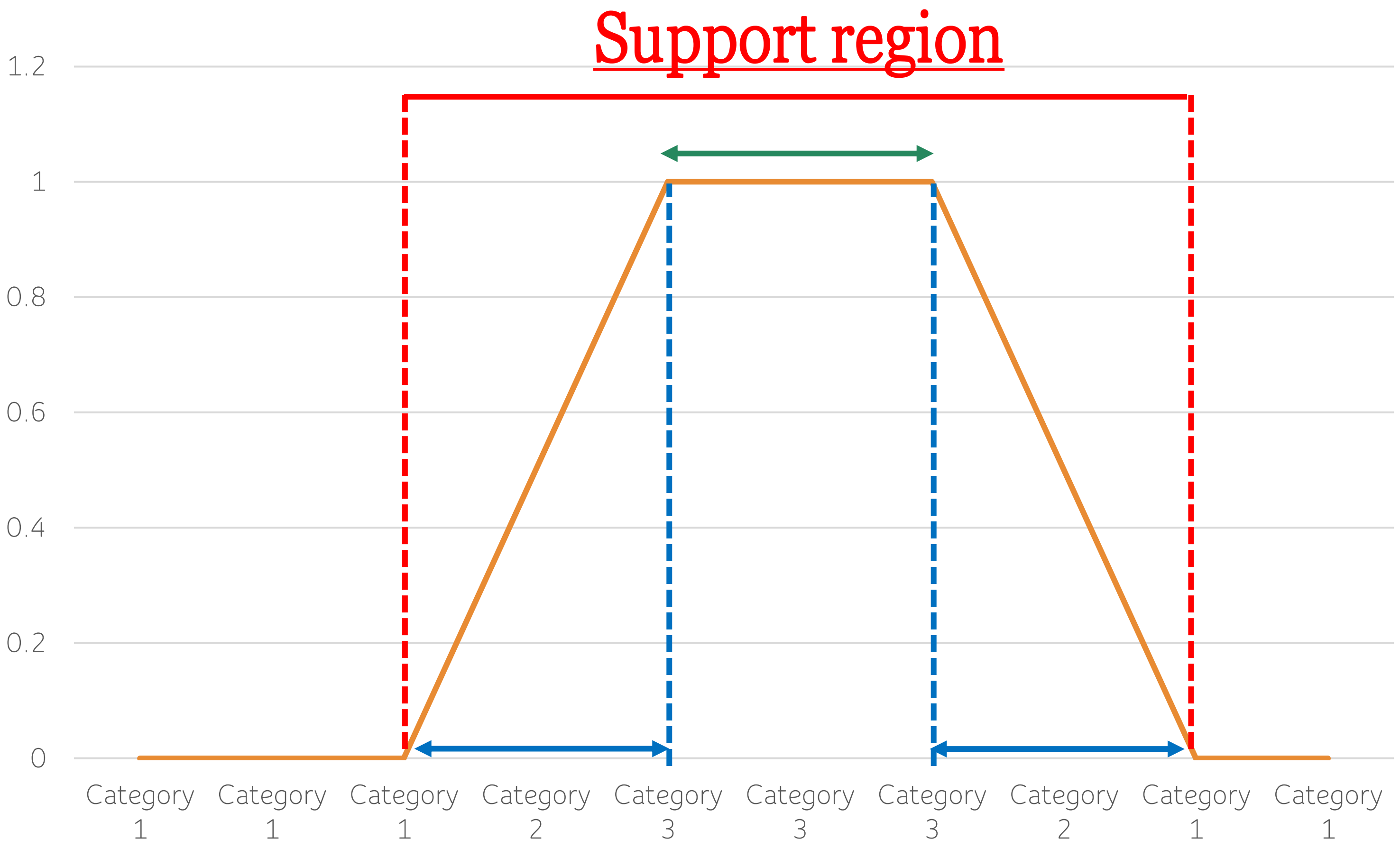
Membership Function of classical set A

•Membership function

•Example: in the fuzzy set *"tall people"*, a person who is 180 cm might belong with a degree of 0.6, while someone 190 cm might have 1.

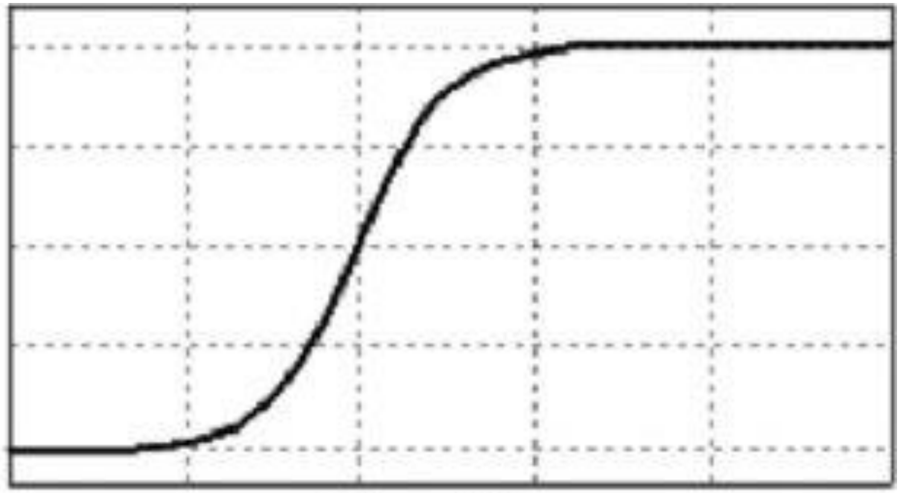


- Who can give us an example that follow the below membership function



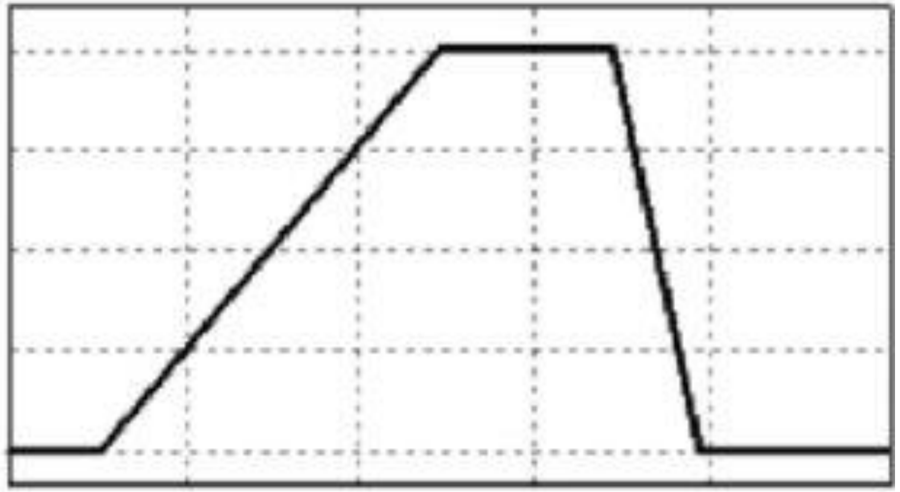
- **Support region** : characterized by nonzero membership $\mu(x)>0$
- **Core region** : characterized by full membership $\mu(x)=1$
- **Boundary region** : characterized by full membership $0<\mu(x)<1$

Gaussian



Sigmoid

Triangular



Trapezoidal

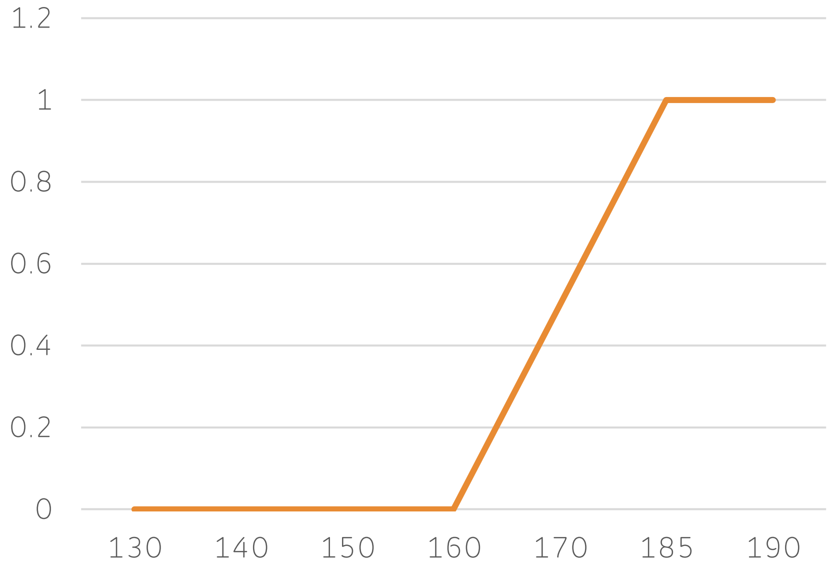
Operations on Fuzzy Sets

Let's consider the following fuzzy sets with their membership function :

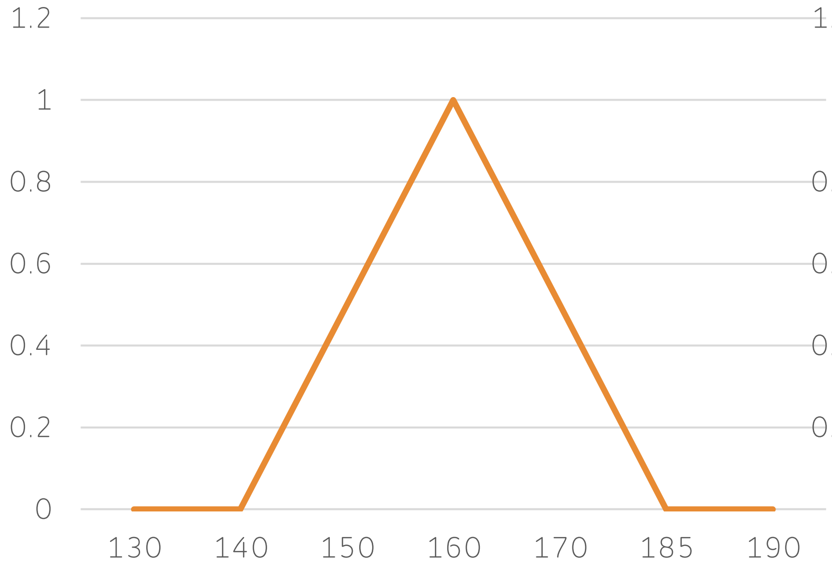
Fuzzy set A : the set of tall people

Fuzzy set B : the set of regular people

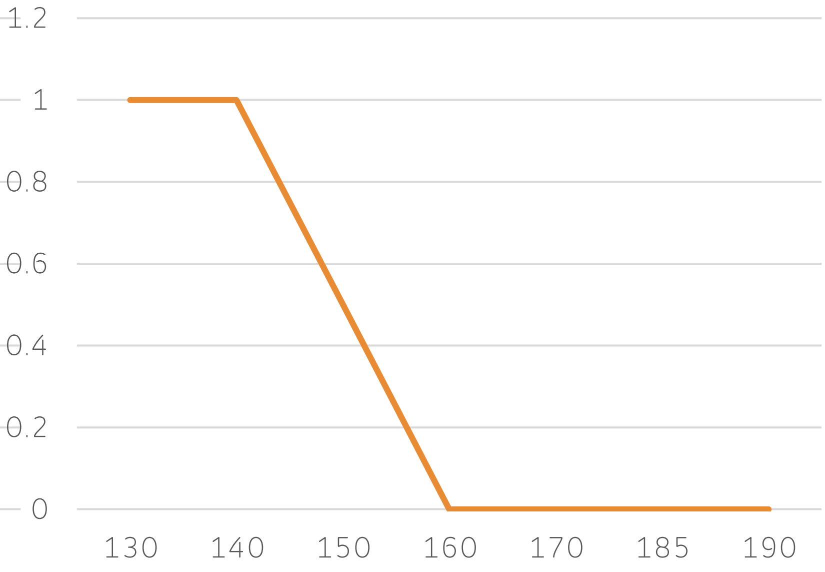
Fuzzy set C : the set of small people



A



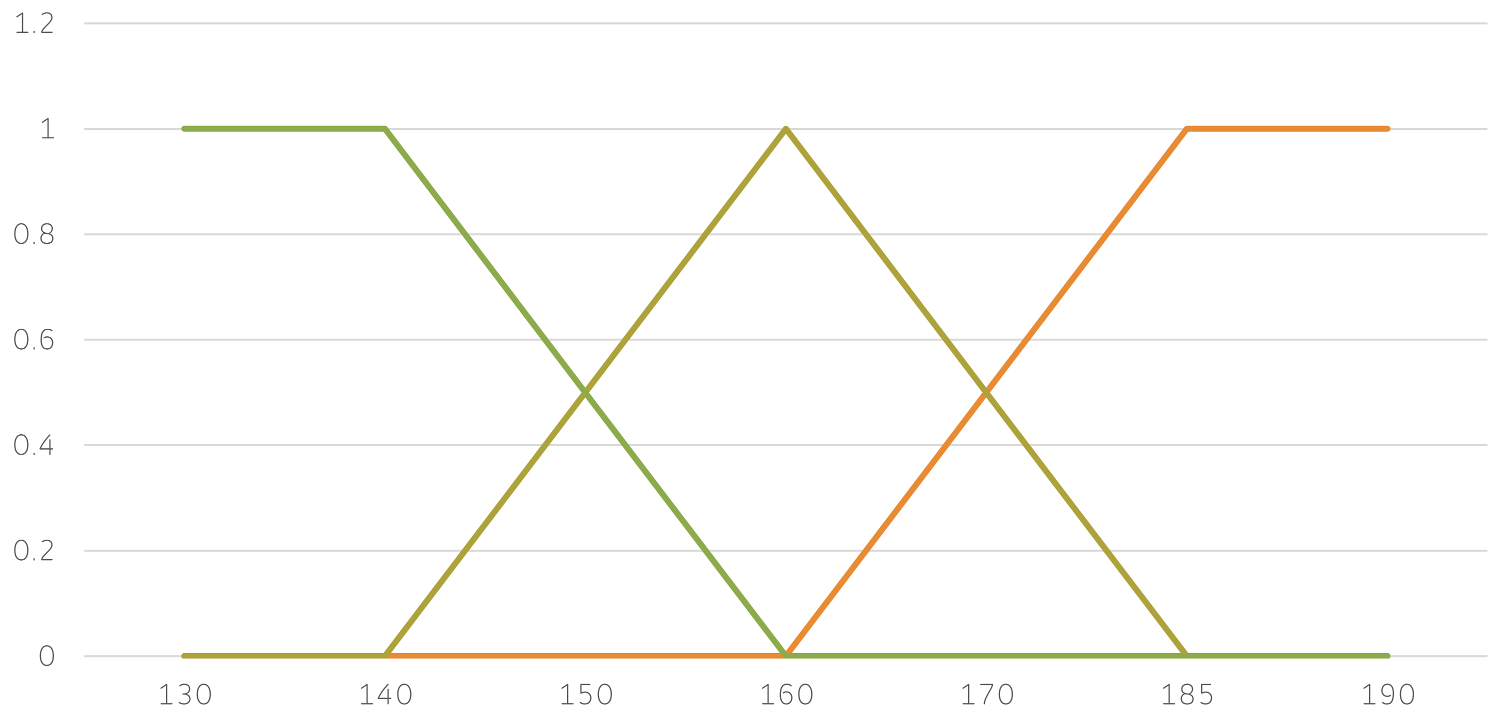
B



C

Operations on Fuzzy Sets

Let's consider the following fuzzy sets with their membership function :



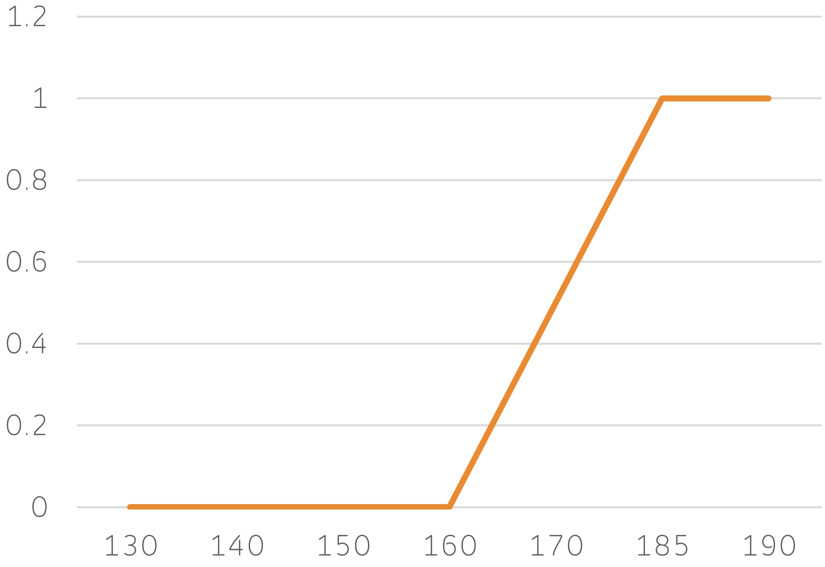
Operations on Fuzzy Sets

Fuzzy Compliment (Fuzzy negation)

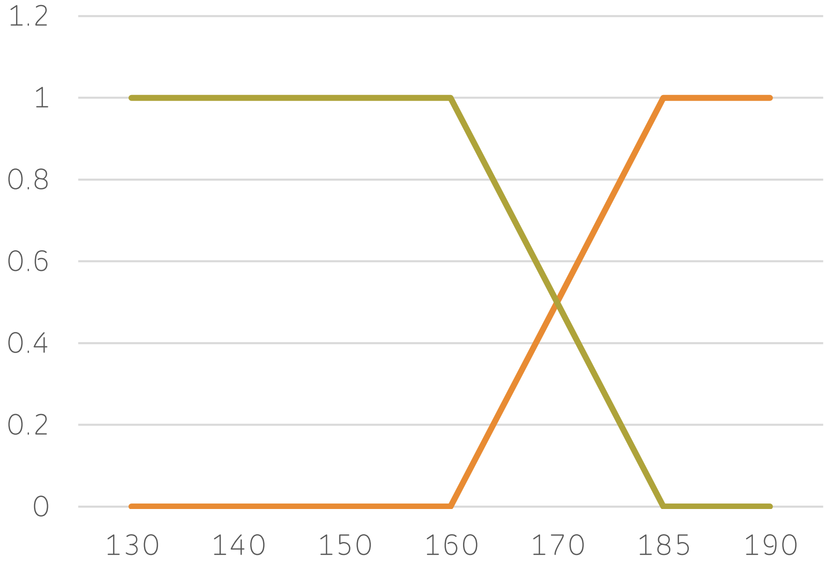
Fuzzy Compliment of the fuzzy set A, is the set of elements of U excluding these of A

$\bar{A} = \{x \mid x \notin A \;\; and \; x \in U\}$ But it is also a fuzzy set $\mu_{\bar{A}}(x) = 1 - \mu_A(x)$

Fuzzy set A : the set of tall people



$\bar{A}$



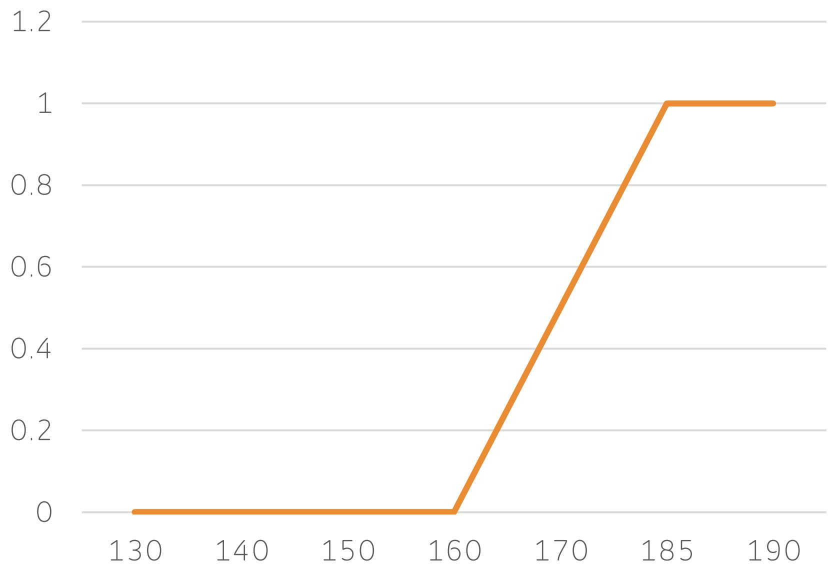
Operations on Fuzzy Sets

Fuzzy Union (Fuzzy OR)

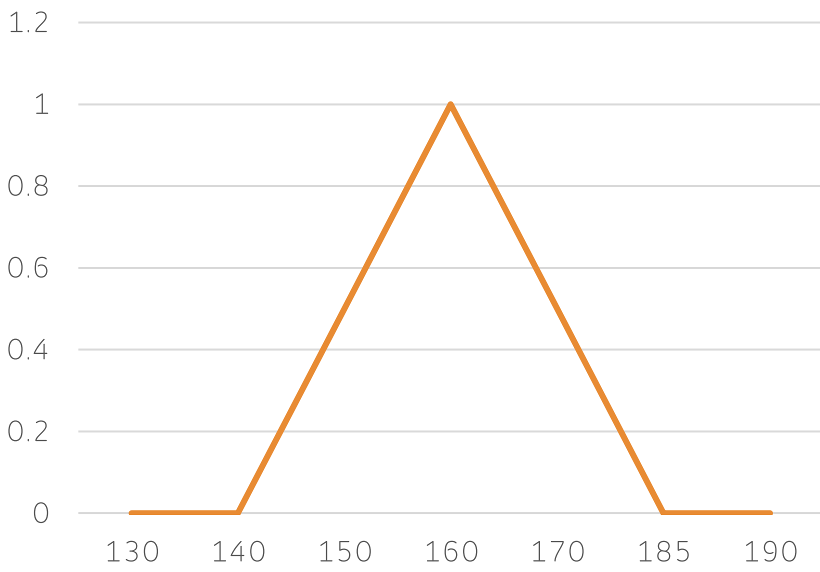
Fuzzy Union of the two fuzzy set A, B is the set of elements of D given by

$$\mu_D(x) = \max\{\mu_A(x), \mu_B(x)\} \quad \forall\, x \in U$$

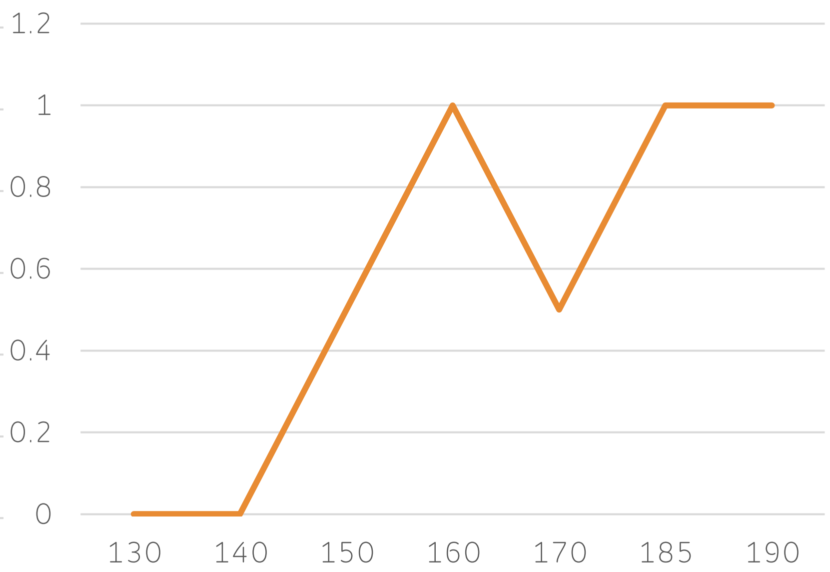
A : the set of tall people



B : the set of regular people



D



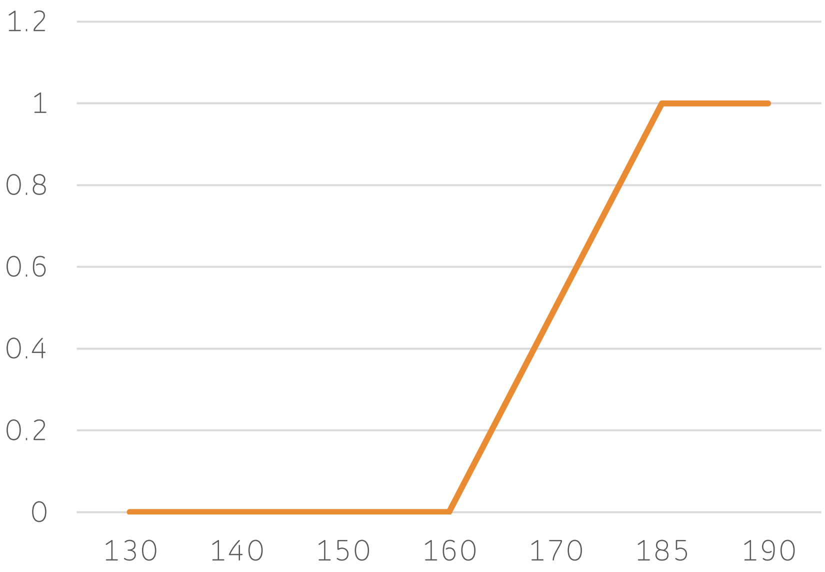
Operations on Fuzzy Sets

Fuzzy Intersection (Fuzzy AND)

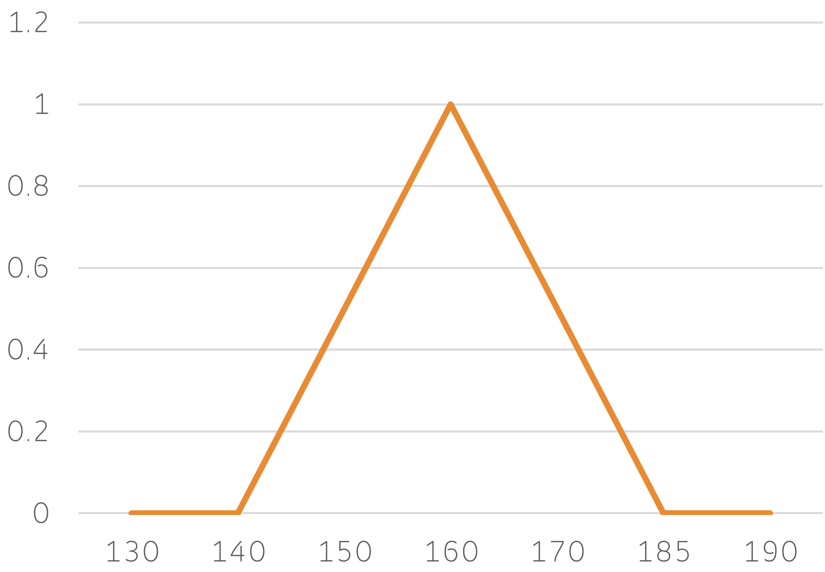
Fuzzy Intersection of the two fuzzy set A, B is the set of elements of D given by

$$\mu_D(x) = \min\{\mu_A(x), \mu_B(x)\} \quad \forall\, x \in U$$

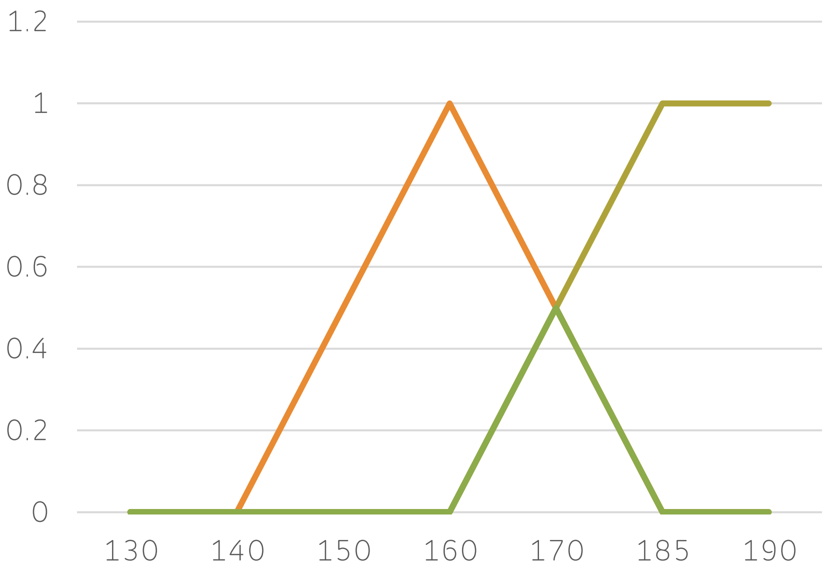
A : the set of tall people



B : the set of regular people



D



Operations on Fuzzy Sets

With the usual definitions of fuzzy operators, we always find the properties of commutativity, distributivity and associativity classics. However, there are two notable exceptions:

- In fuzzy logic, the law of excluded middle is contradicted:

$$A \cup \bar{A} \neq U$$

- In fuzzy logic, an element can belong to A and not A at the same time:

$$A \cap \bar{A} \neq \emptyset$$

Linguistic variables

Formal Definition

A linguistic variable is defined as a variable that can take linguistic values, which are described by fuzzy sets on a universe of discourse.

Example: Let's suppose we want to solve a problem related with temperature

Linguistic variable:
Temperature

The universe
U = {0….100}

Linguistic values

Cold : {0..25}

warm : {20..30}

Hot : {30..40}

Linguistic variables

Let’s consider a system to control a fan installed in small room, the system take two inputs (temperature, humidity) and one single output which is the speed of the fan.

Here we have three linguistic variables which are : temperature, humidity and speed, each one has it own linguistic values

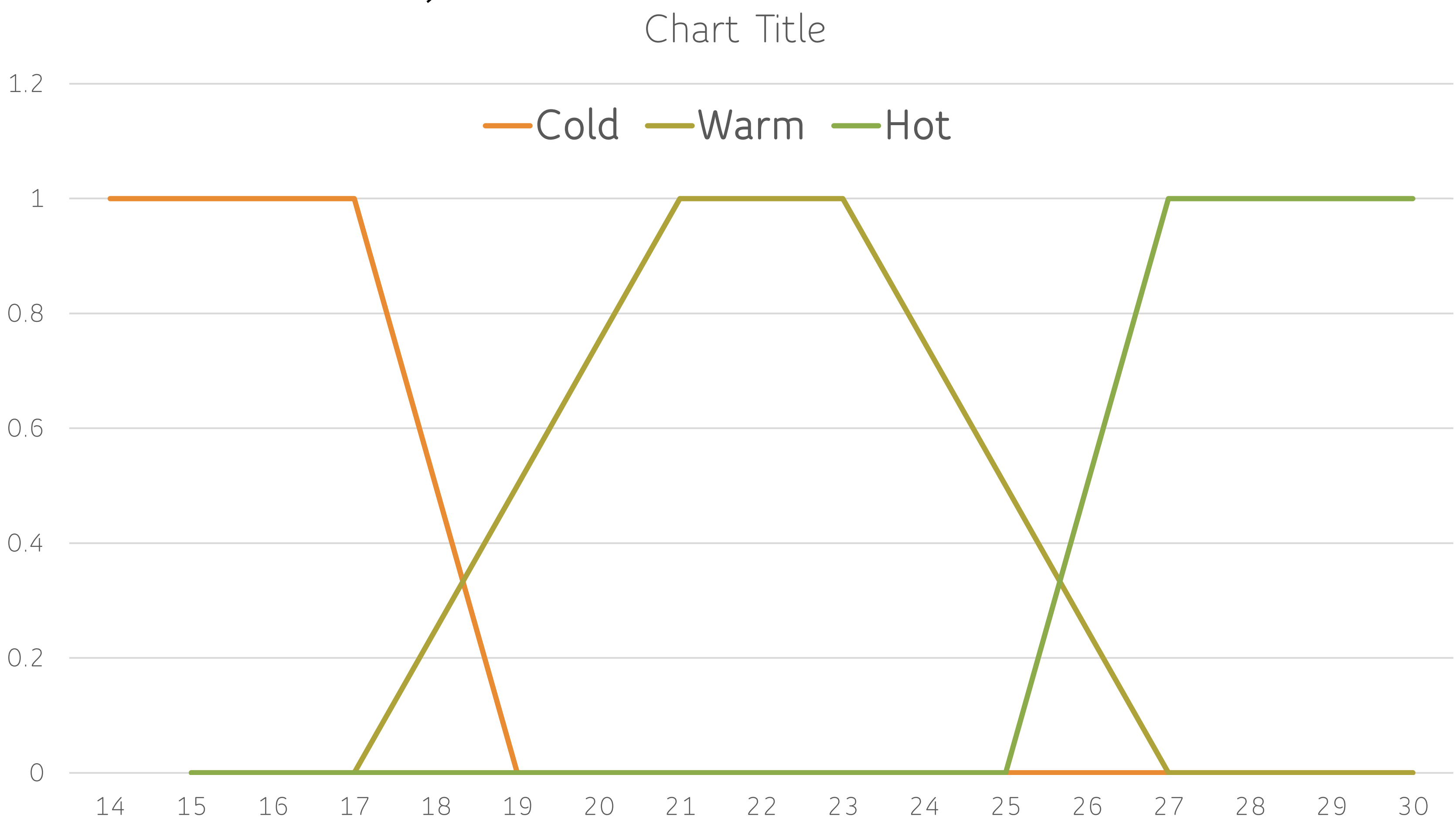
Temperature could be : Cold, Warm and Hot.

Humidity could be : dry and wet.

Speed could be : low, average and high

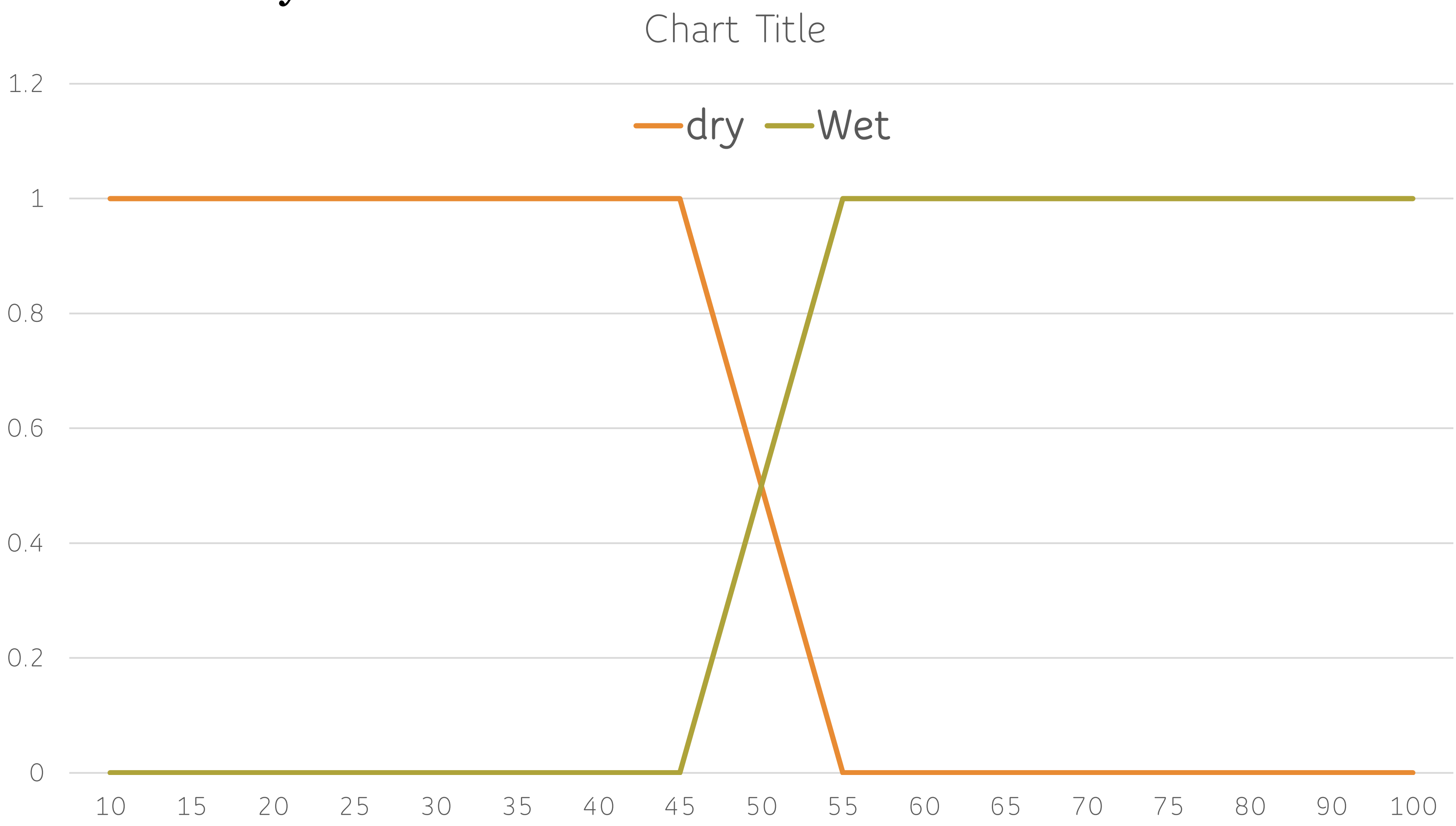
Linguistic variables

Temperature could be : Cold, Warm and Hot.



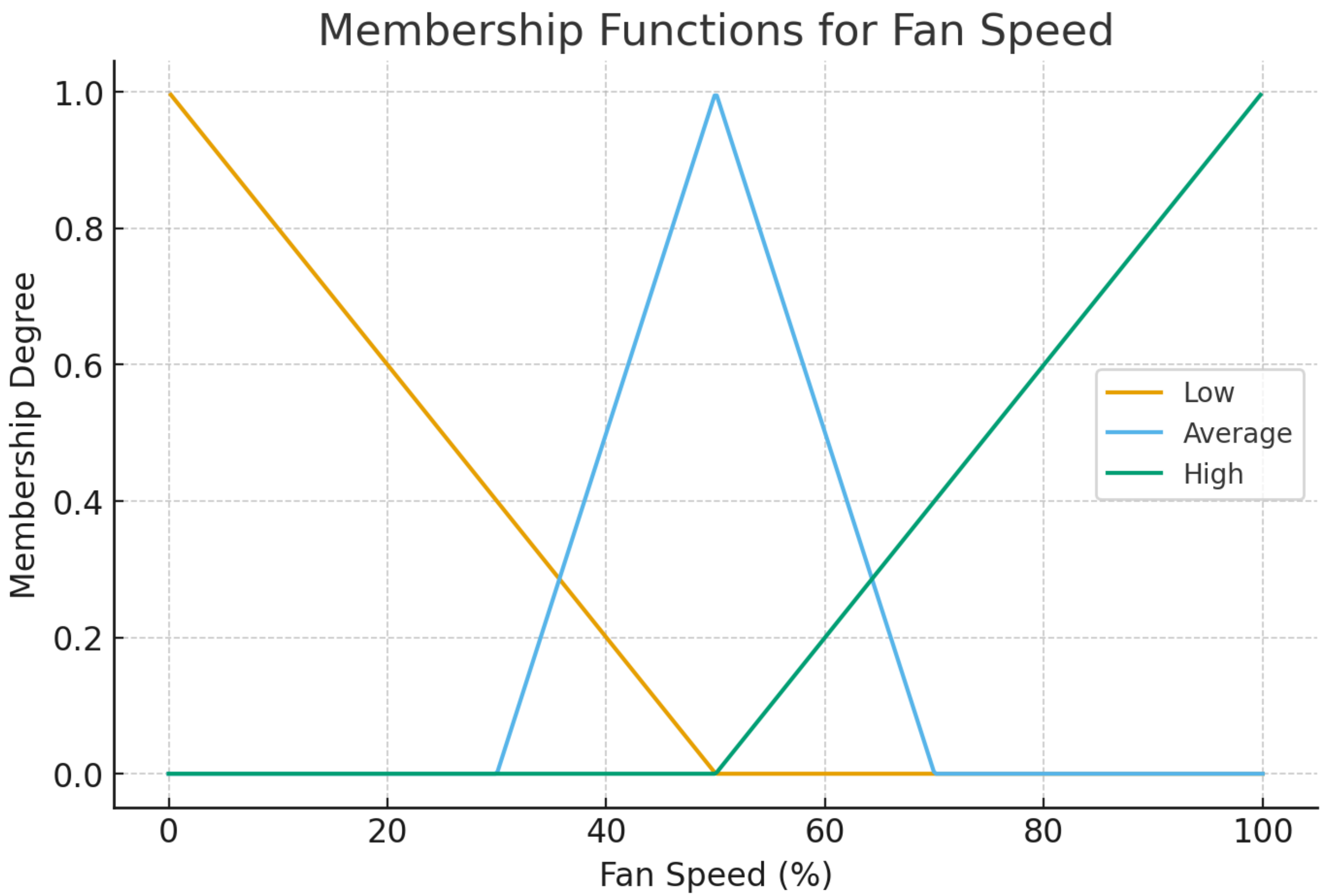
Linguistic variables

Humidity could be : dry and wet.



Linguistic variables

Speed could be : low, average and high



Inference rules

Example of inference rules

R1: IF Temperature IS Cold AND Humidity IS Dry THEN FanSpeed IS Low

R2: IF Temperature IS Cold AND Humidity IS Wet THEN FanSpeed IS Average

R3: IF Temperature IS Warm AND Humidity IS Dry THEN FanSpeed IS Average

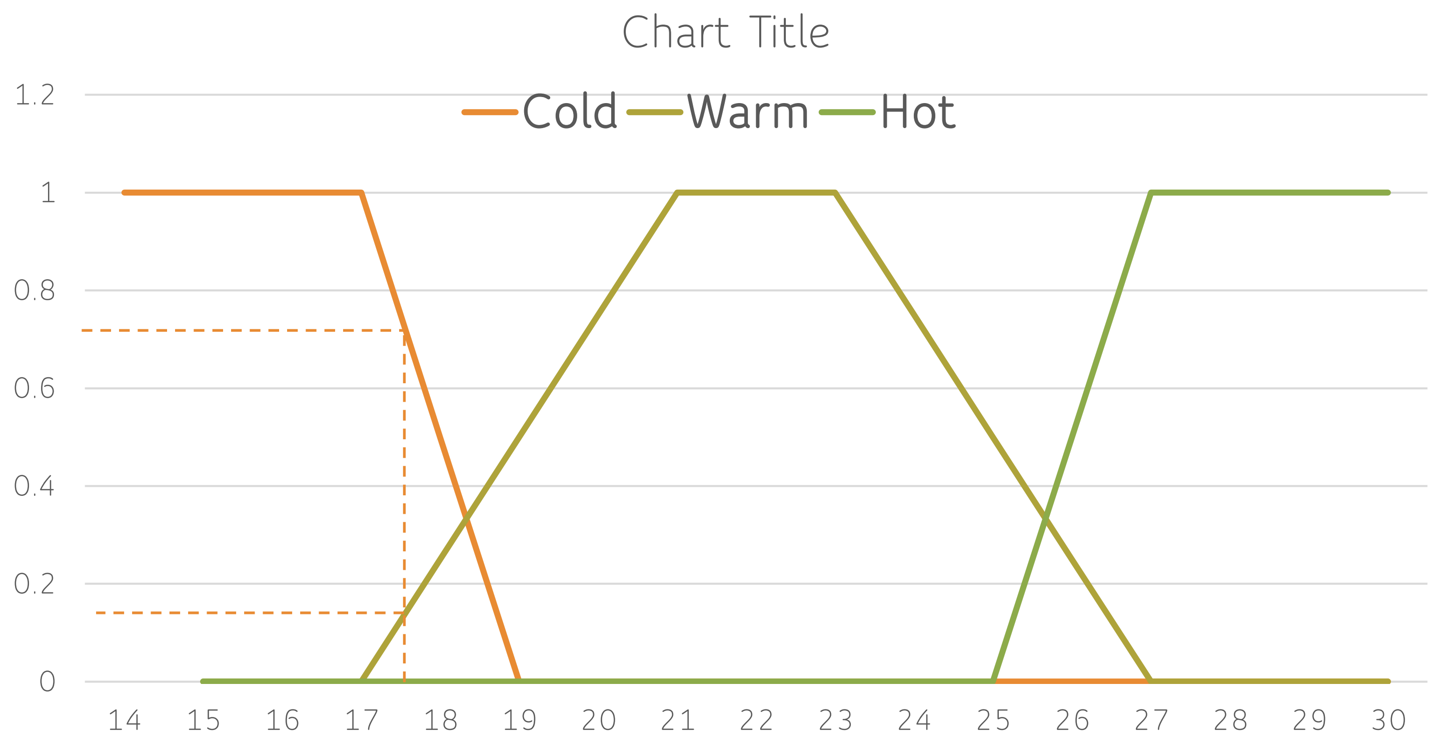
R4: IF Temperature IS Warm AND Humidity IS Wet THEN FanSpeed IS High

R5: IF Temperature IS Hot AND Humidity IS Dry THEN FanSpeed IS High

R6: IF Temperature IS Hot AND Humidity IS Wet THEN FanSpeed IS High

Inference rules

Example of using the inference rules



Inputs (crisp values)

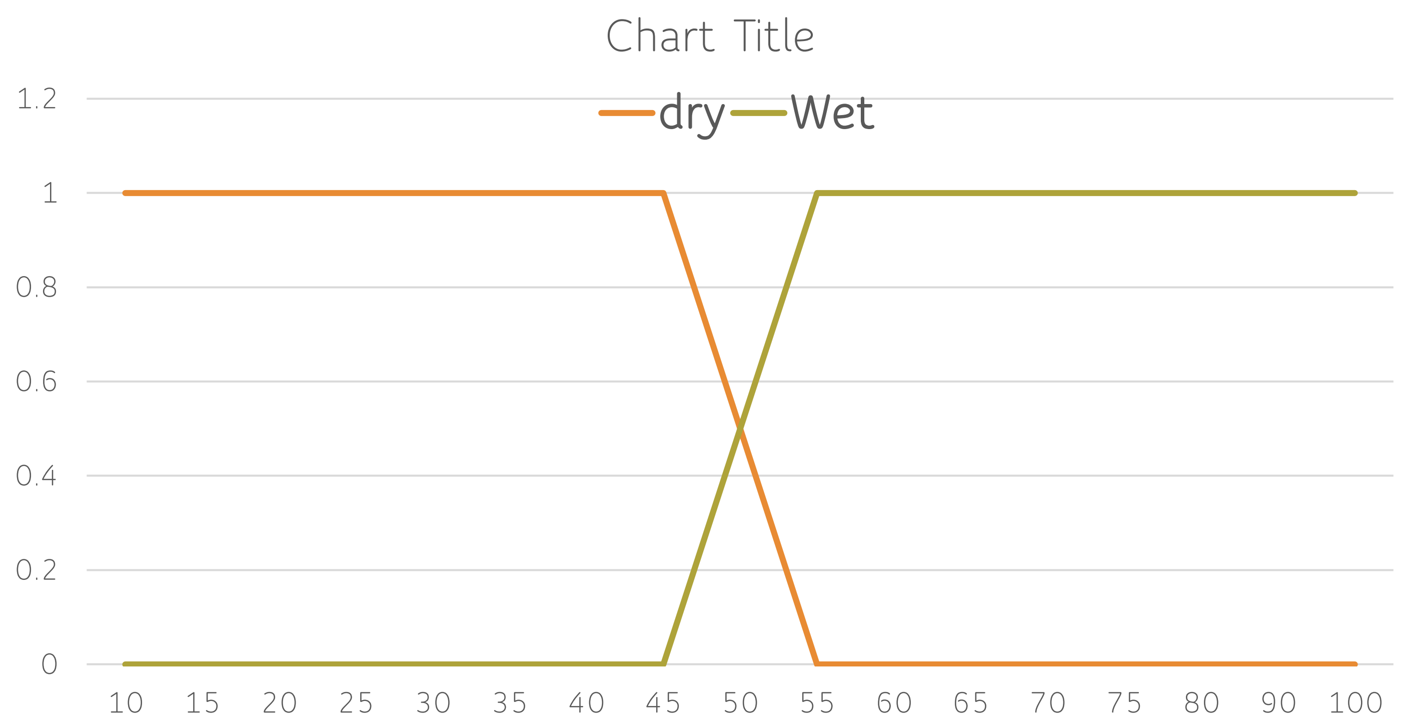
- Temperature = **16 °C**
- Humidity = **70 %**

Inputs (membership value)

- Temperature = **16 °C**
  - **Cold = 0.7**
  - **Warm = 0.1**
  - **Hot = 0**

Inference rules

Example of using the inference rules



Inputs (crisp values)

- Temperature = 16 °C
- Humidity = 70 %

Inputs (membership value)

- Temperature = 16 °C
  - Cold = 0.7
  - Warm = 0.1
  - Hot = 0

Inputs (membership value)

- Humidity = 70 %
  - Dry = 0
  - Wet =1

Inference rules

Example of using the inference rules

Inputs (membership value)

- Temperature = **16 °C**
  - **Cold = 0.7, Warm = 0.1, Hot = 0**
- Humidity = **70 %**
  - **Dry = 0, Wet =1**

R1: **IF** Temperature **IS** Cold **AND** Humidity **IS** Dry **THEN** FanSpeed **IS** Low

Min(0.7, 0) $\rightarrow$ Firing Strength =0

R2: **IF** Temperature **IS** Cold **AND** Humidity **IS** Wet **THEN** FanSpeed **IS** Average

Min(0.7, 1) $\rightarrow$ Firing Strength =0.7

R3: **IF** Temperature **IS** Warm **AND** Humidity **IS** Dry **THEN** FanSpeed **IS** Average

Min(0.1, 0) $\rightarrow$ Firing Strength =0.1

Inference rules

Example of using the inference rules

Inputs (membership value)

- Temperature = **16 °C**
  - **Cold = 0.7, Warm = 0.1, Hot = 0**
- Humidity = **70 %**
  - **Dry = 0, Wet =1**

R4: **IF** Temperature **IS** Warm **AND** Humidity **IS** Wet **THEN** FanSpeed **IS** High

Min(0.1, 1) $\rightarrow$ Firing Strength =0.1

R5: **IF** Temperature **IS** Hot **AND** Humidity **IS** Dry **THEN** FanSpeed **IS** High

Min(0, 0) $\rightarrow$ Firing Strength =0

R6: **IF** Temperature **IS** Hot **AND** Humidity **IS** Wet **THEN** FanSpeed **IS** High

Min(0, 1) $\rightarrow$ Firing Strength =0

Inference rules

Example of using the inference rules

Inputs (membership value)

- Temperature = **16 °C**
  - **Cold = 0.7, Warm = 0.1, Hot = 0**
- Humidity = **70 %**
  - **Dry = 0, Wet =1**

R2: **IF** Temperature **IS** Cold **AND** Humidity **IS** Wet **THEN** FanSpeed **IS** Average

Min(0.7, 1) → Firing Strength =0.7

R3: **IF** Temperature **IS** Warm **AND** Humidity **IS** Dry **THEN** FanSpeed **IS** Average

Min(0.1, 0) → Firing Strength =0.1

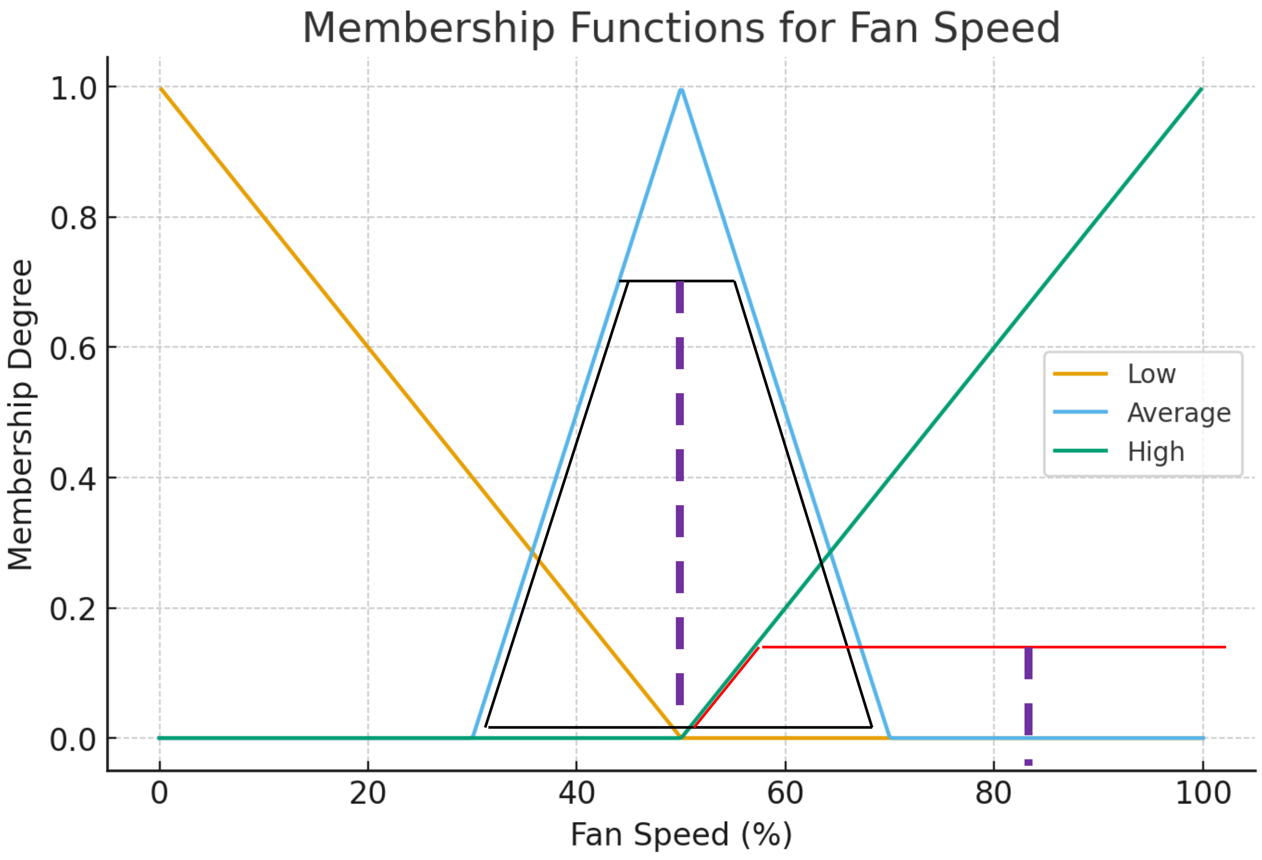
R4: **IF** Temperature **IS** Warm **AND** Humidity **IS** Wet **THEN** FanSpeed **IS** High

Min(0.1, 1) → Firing Strength =0.1

Here we take High =0.1, and Average =0.7 (we took the max)

Inference rules

Here we take High =0.1, and Average =0.7 (we took the max)

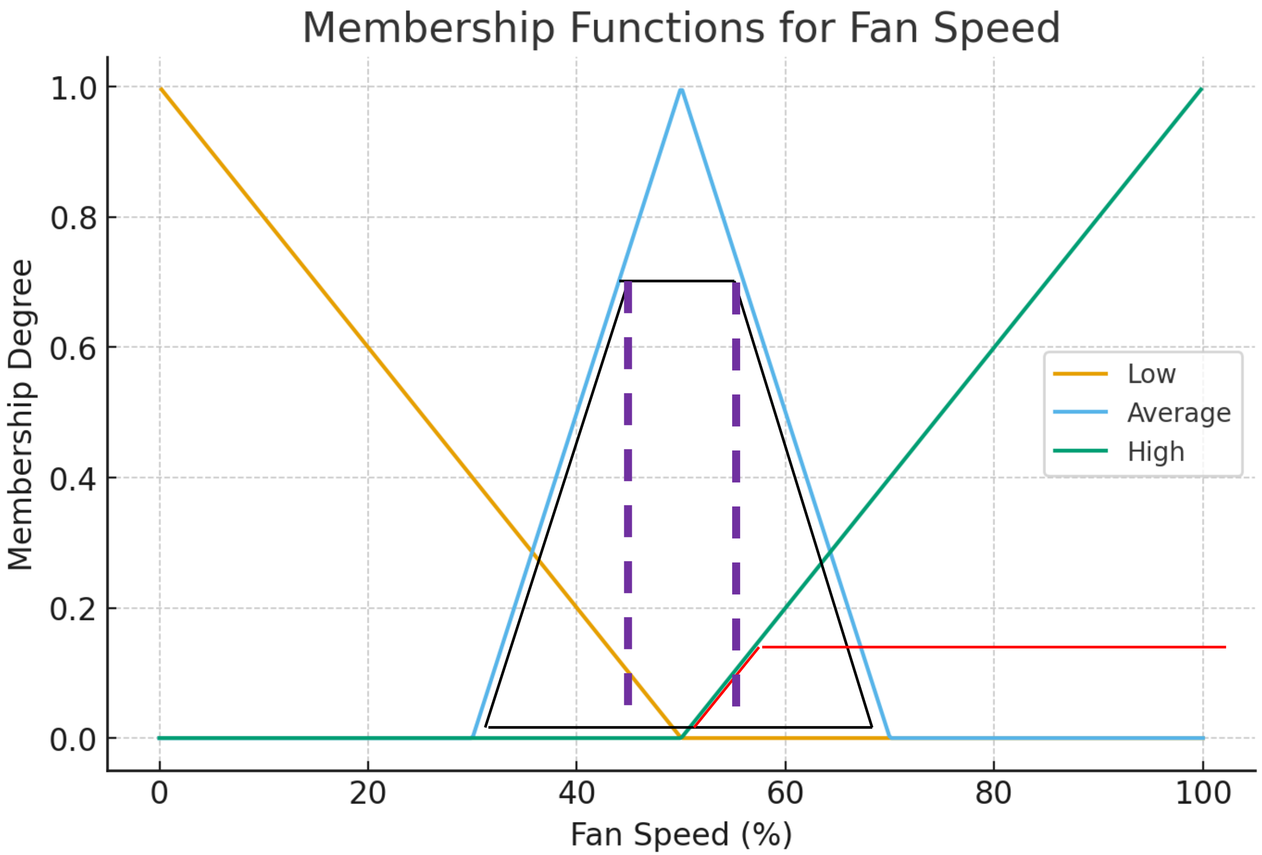


Weighted average method

$$\frac{(50 * 0.7) + (82.5 * 0.1)}{0.1 + 0.7} = 54.06 \approx 55$$

Inference rules

Here we take High =0.1, and Average =0.7 (we took the max)



- Maxima Methods
- (Mean of Maxima Method (MOM))

$$\frac{(45 + 55)}{2} = 50$$

- Center of Sums Method (COS)
- Center of gravity (COG) / Centroid of Area (COA) Method
- Center of Area / Bisector of Area Method (BOA)
- Weighted Average Method
- Maxima Methods

Inference system

Example of using the inference rules