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Master 2 IDO S3 2024/2025
Optimization Methods / WRITTEN EXAMINATION (Duration 30')

We aim to maximize the function f with two integer variables x, y such that:
$f(x,y) = x^2 + 2y + 1$ using a genetic algorithm.

1) **Assume the current population consists of 4 individuals: a = (4,3); b = (5,5); c = (0,6); d = (2,0).**
   a) **Calculate the probabilities of selection by roulette then by rank of each of these individuals.**
   b) **How to interpret the difference between these probabilities?**
   c) **Suppose each integer is encoded using 4 bits (a solution is then encoded using 8 bits).**
      Calculate the fitness of the one-point crossover results of solutions a and b.
      Compute the fitness of the mutation result of a which consists of inverting a bit.
2) **Assume a population of m individuals and each solution is encoded using n bits.**
   Write the crossover and mutation algorithms defined above.

Answer :

1) a) Calculate Probabilities of Selection

Using Roulette Wheel Selection

The probability of selection is proportional to the fitness of each individual.
Fitness function: $f(x,y) = x^2 + 2y + 1$
Fitness Calculations: f(a) = 23 ; f(b) = 36 ; f(c) = 13 ; f(d) = 5 , Total Fitness = 77
Probabilities: P(a)=23/77 ≈ 0.2987 , P(b)=36/77 ≈ 0.4675 , P(c)=13/77 ≈ 0.1688 , P(d)=5/77 ≈ 0.0649.

Using Rank-Based Selection

Rank individuals based on fitness (lowest to highest): d<c<a<b , **Ranks**: a:3, b:4, c:2, d:1
Assign probabilities based on ranks : P(b)=4/10=0.4, P(a)=3/10=0.3, P(c)=2/10=0.2, P(d)=1/10=0.1

b) Interpretation of Differences

- **Roulette Selection**: Reflects the absolute fitness values. Stronger solutions are more likely to be selected, but weaker solutions still have a chance proportional to their fitness.
- **Rank Selection**: Reduces the bias of absolute fitness differences. It prioritizes relative fitness, ensuring diversity and avoiding domination by very fit individuals.

c) Fitness of One-Point Crossover (a, b)

Encoding a=(4,3) , b=(5,5)

x=4 = 0100, y=3 = 0011 , a = 01000011
x=5 = 0101, y=5 = 0101 , b = 01010101

One-Point Crossover: Assume crossover point after 5th bit.

a′ = 01000|101 = (4, 5)
b′ = 01010|011 = (5, 3)

Fitness:
f(a′) = 27, f(b′) = 32

c) Fitness of Mutation of a

Mutation involves inverting a random bit. Suppose the 2nd bit is flipped.
Original a= 01000011 → Mutated a= a’ = 00000011 = (0,3) , f(a’) = 7

2) Crossover and Mutation Algorithms

```
void crossover(){
   for(i=0;i<m-1;i+=2)
       if (rand()< cp)
          { ind= int(rand()*n);
            for (j=ind;j<n;j++)
                swap(pop[i][j],pop[i+1][j])}}
```

```
void mutation(){
   for (i=0 ; I < m ; i++)
       if (rand()< mp)
          { ind= int(rand()*n);
            pop[i][ind] = 1- pop[i][ind])}}
```