Mohamed Boudiaf University of M'sila
Faculty of Mathematics and Computer Science
Department: Computer Science
Module:Graph Theory/DCSO

Resp.Module: Dr. Said GADRI

TUTORIALS – CORRECTED SERIES NO. 03

EXERCISE N° 01

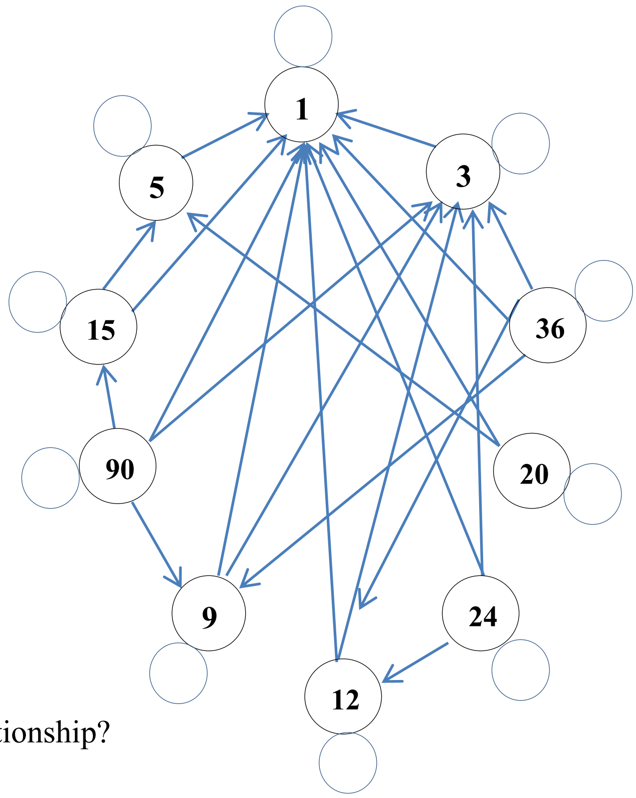
The characteristics of each of the given graphs (main characteristics):
(a) Is a symmetric graph, but it is not complete, and is not a simple graph
(b) Is not symmetric, is not antisymmetric, is not simple
(c) Is not reflexive, is not symmetric, is not simple, is not antisymmetric
(d) Is a simple, antisymmetric, transitive graph, is not reflexive.
(e) A bipartite graph
(f) A reflexive graph
(g) A trivial graph
(h)/(c): (h) is the inverse graph of graph (c)
(i)/(b): (i) is the complementary graph of graph (b)

EXERCISE N° 02

Let be the graph G(X, U) such that:

X = {1, 3, 5, 12, 20, 9, 15, 24, 36, 90}   U = { (x, y) / (x, y) ∈U if x is multiple of y}

1. Plot the graph G.



2. Study the characteristics of G
   - Reflective
   - Antisymmetric
   - Transitive
3. What can be said about the “multiple” relationship?

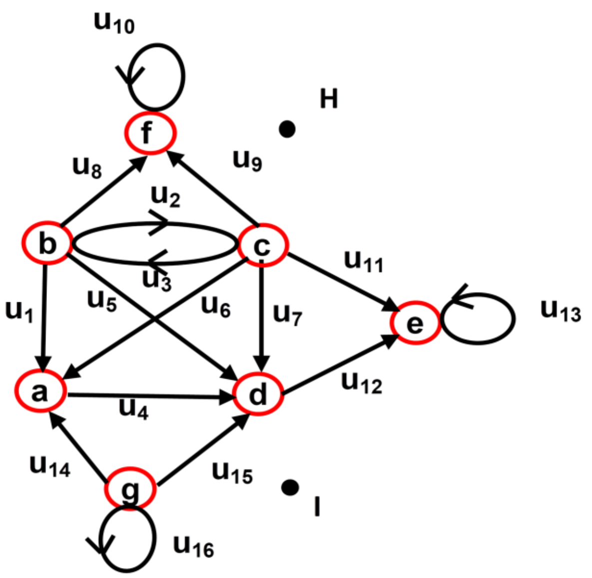
   Is a relation: reflexive, antisymmetric, transitive ==> is an order relation

4. Are there any circuits in G?

   There are no circuits in G

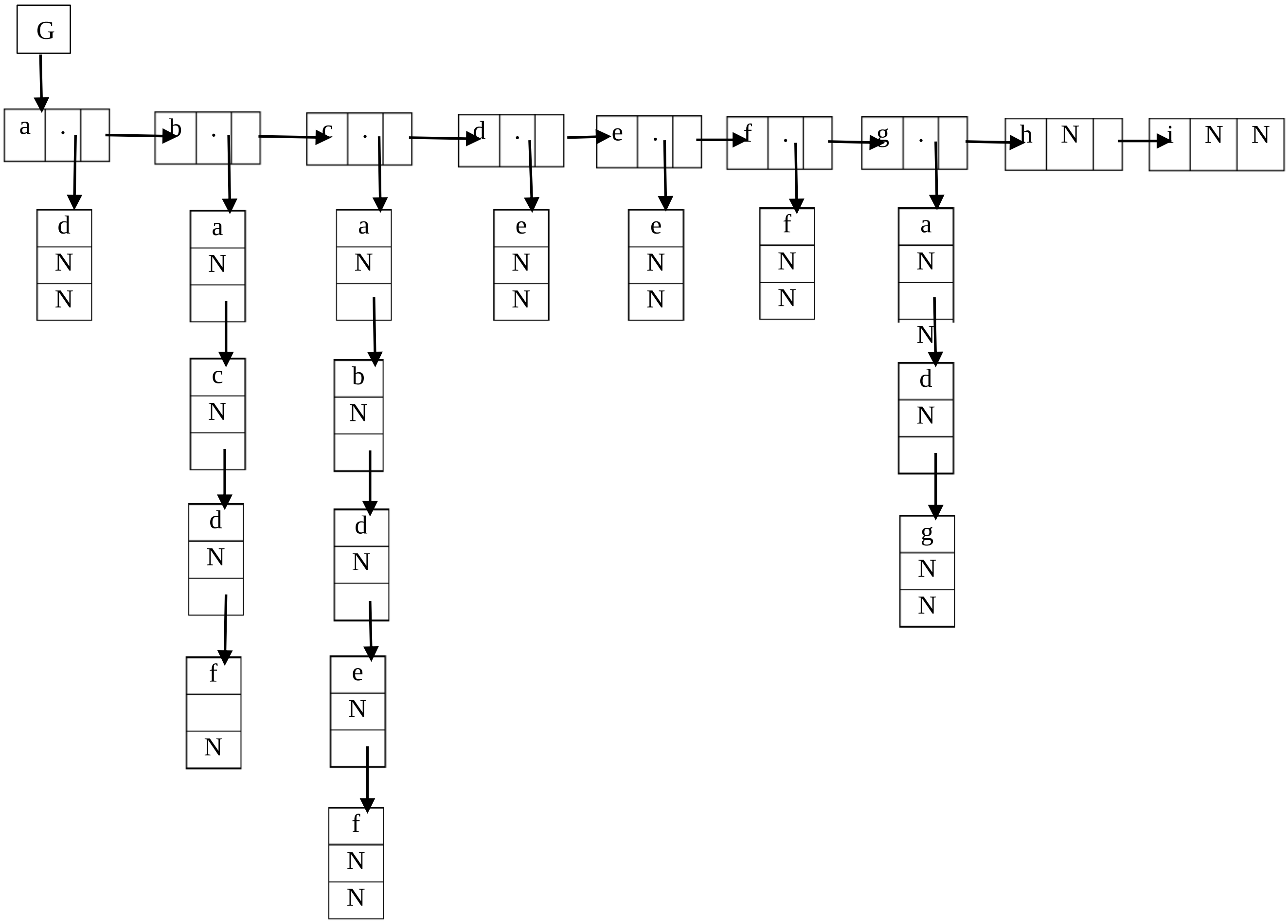
<u>EXERCISE N° 03</u>

Given the following graph G(X, U):

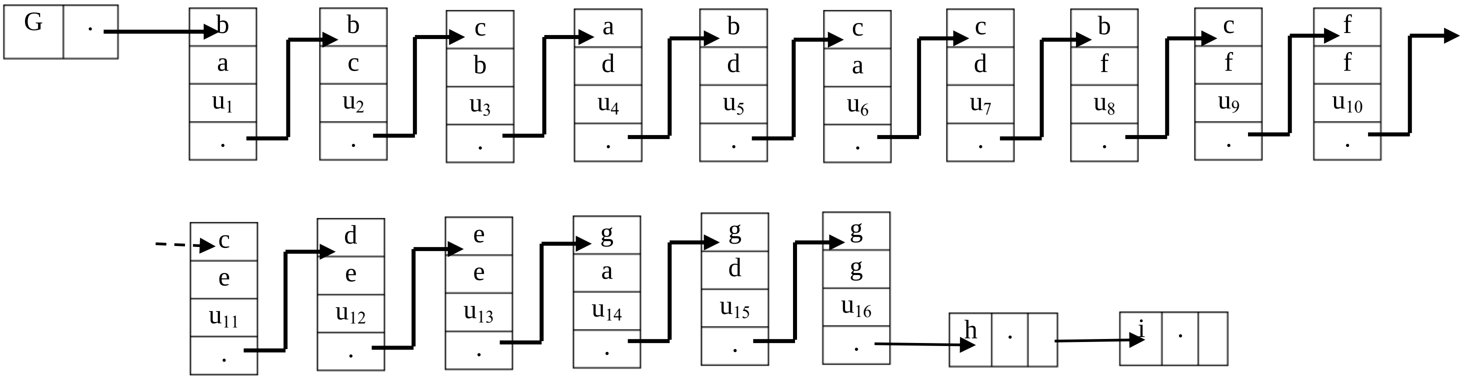


1. Represent the graph using a dynamic methods following:

   a) Dynamic list of vertices



b) Dynamic list of arcs



Sommets isolés

2. Is the graph G simply connected?
Not Simply connected

3. Is it strongly connected?
Not Strongly connected

4. Give the simply connected components SCC of G
{a, b, c, d, e, f, g}, {h}, {i}

5. Give the strongly connected components Str.CC of G
{a}, {b, c}, {d}, {e}, {f}, {g}

EXERCISE N° 04

A company directors' board is composed of nine (09) members: Mrs. A, B, C, D, E, F and Ms. G, H and I. Each of these people influences a certain number of their colleagues as shown in the table below:

Mr or Mrs	Influence
A	B, C, F
B	C, E, I
C	E, G, D
D	A, B, E, F, G, I
E	None
F	C, E
G	A, B, E
H	A, C, E
I	G, H

1. Represent the influence game within this council using a graph

   It is a directed graph

2. Give the adjacency matrix of the resulting graph.

   A simple matrix (vertex – vertex)

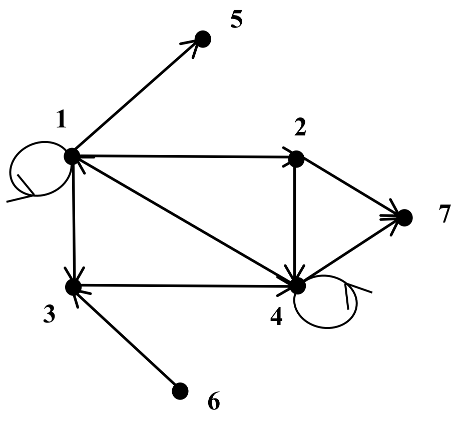
EXERCISE N° 05

Let the following two matrices defined as follows:

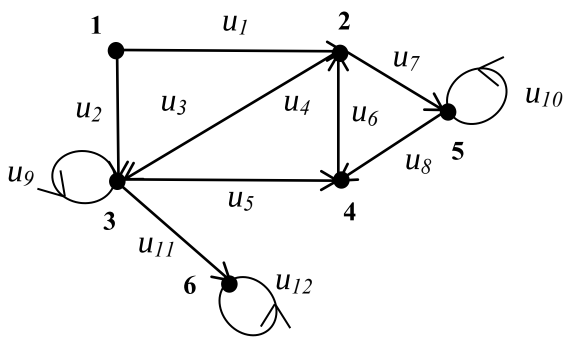
	1	**2**	**3**	**4**	**5**	**6**	**7**
1	1	1	1	0	1	0	0
2	0	0	0	1	0	0	1
3	0	0	0	1	0	0	0
4	1	0	0	1	0	0	1
5	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0
7	0	0	0	0	0	0	0

	u1	**u2**	**u3**	**u4**	**u5**	**u6**	**u7**	**u8**	**u9**	**u10**	**u11**	**u12**
1	+1	+1	+1	0	0	0	0	0	0	0	0	0
2	-1	0	0	+1	0	-1	+1	0	0	0	0	0
3	0	-1	0	-1	+1	0	0	0	2	0	+1	0
4	0	0	-1	0	-1	+1	0	-1	0	0	0	0
5	0	0	0	0	0	0	-1	+1	0	2	0	0
6	0	0	0	0	0	0	0	0	0	0	-1	2

1. Plot the graphs corresponding to the previous matrices



(G1)



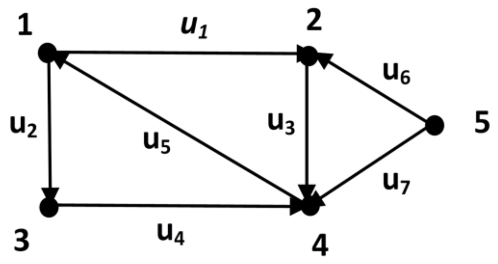
(G2)

2. Are the resulting graphs connected? Are they strongly connected?

   Graphs G1, G2 are connected, not strongly connected

EXERCISE N° 06

Let G(X, U) be a graph with:
X = {1, 2, 3, 4, 5}; U = {u1, u2, u3, u4, u5, u6, u7}



1. Give the adjacency matrix M(G)

$$M(G) = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

2. Give the path length matrix: $M^{(2)}$

The matrix of number of paths of length 2 connecting xi and xj is:

$$M^2 = MxM = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} x \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{2} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \end{bmatrix}$$

3. Give the distance matrix D

For $x, y \in X$, $d(x, y) = \begin{cases} \infty \ \nexists \, path\ from\ x\ to\ y \\ Min[l(c)]\ with\ l(c)\ a\ path\ from\ x\ to\ y \\ 0\ \ Otherwise \end{cases}$

So

$$D(G) = \begin{bmatrix} \infty & \mathbf{1} & \mathbf{1} & \mathbf{2} & \infty \\ \mathbf{2} & \infty & \mathbf{3} & \mathbf{1} & \infty \\ \mathbf{2} & \mathbf{3} & \infty & \mathbf{1} & \infty \\ \mathbf{1} & \mathbf{2} & \mathbf{2} & \infty & \infty \\ \mathbf{2} & \mathbf{1} & \mathbf{3} & \mathbf{1} & \infty \end{bmatrix}$$

GOOD LUCK