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Faculty of Mathematics and Computer Science

Department: Computer Science

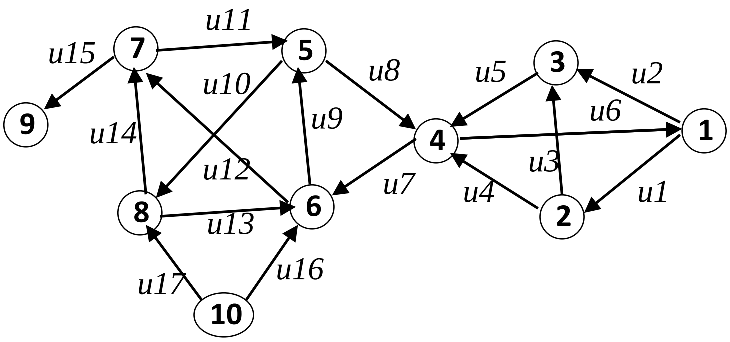
Module:Graph Theory/DCSO

TUTORIALS - - SERIES NO. 01 CORRECTED

Exercise 01

1. Graph order: n = |X| = 10

   Graph size: m = |U| = 17



2. Representation of the graph using:

a) An adjacency matrix (vertex-vertex)

	1	2	3	4	5	6	7	8	9	10
1	0	1	1	0	0	0	0	0	0	0
2	0	0	1	1	0	0	0	0	0	0
3	0	0	0	1	0	0	0	0	0	0
4	1	0	0	0	0	1	0	0	0	0
5	0	0	0	1	0	0	0	1	0	0
6	0	0	0	0	1	0	1	0	0	0
7	0	0	0	0	1	0	0	0	1	0
8	0	0	0	0	0	1	1	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	1	0	1	0	0

b) An incidence matrix (vertex – arcs)

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}	u_{13}	u_{14}	u_{15}	u_{16}	u_{17}
1	+1	+1	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0
2	-1	0	+1	+1	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	-1	-1	0	+1	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	-1	-1	+1	+1	-1	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	+1	-1	+1	-1	0	0	0	0	0	0
6	0	0	0	0	0	0	-1	0	+1	0	0	+1	-1	0	0	-1	0
7	0	0	0	0	0	0	0	0	0	0	+1	-1	0	-1	+1	0	0
8	0	0	0	0	0	0	0	0	0	-1	0	0	+1	+1	0	0	-1
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	+1

3. $\Gamma^+(6) = \{5, 7\}$, $\Gamma^-(4)$, $= \{2, 3, 5\}$, $\Gamma(5) = \{4, 6, 7, 8\}$.
4. $d^+(8) = 2$, $d^-(2) = 1$, $d(6) = 5$.
5. $I^+(4) = \{u_6, u_7\}$, $I^-(6) = \{u_7, u_{13}, u_{16}\}$, $I(8) = \{u_{10}, u_{13}, u_{14}, u_{17}\}$.

6. Find a path between vertices 4 and 6, is it simple? Is it elementary?
   C1 = (4, 6) simple and elementary path
   C2 = (4, 6, 7, 5, 8, 6) simple but not elementary path.
7. Give a simply connected component
   CSC 1 = {1, 2, 3, 4}; CSC 2 = {5, 6, 7, 8, 9, 10}

8. Give a strongly connected component
   CFC 1 = {1, 2, 3, 4}; CFC 2 = {4, 5, 6, 7, 8}

Remove the orientation of the graph and calculate:

1. The density D of the graph G

   For an undirected graph like the given graph, we calculate the maximum size of the graph as follows:

$$Lmax = N*(N-1)/2 = 10*(10 - 1)/2 = 45$$

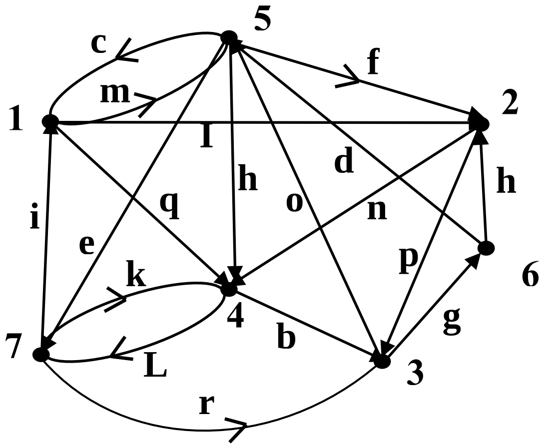
   The density value is then: Density = L/ Lmax = 17/45≈0.378

2. Give a community of vertices
   C1 = {1, 2, 3, 4}; C2 = {5, 6, 7, 8}

3. Is there a bridge? Determine it?
   Yes, there is a bridge which is an edge whose deletion disconnects the graph, it is the edge $\{u_{15}\}$, because if we delete it we will have vertex 9 isolated.

Exercise 02

We have the graph G(X, U) with:
X = {1, 2, 3, 4, 5, 6, 7}
U={a, b, c, d, e, f, g, h, i, j, k, l, m, n}



1 - Among the n-uples below we specify: paths, simple paths, elementary paths:
   (g, d, f, i): is not a path (non-continuous trajectory)
   (h, b, e, j): is not a path (non-continuous trajectory)
   (b, g, d, c, m, e, k, b, g, a): a path, is not simple, is not elementary
   (i, q, b, g, d, c, j): a simple but non-elementary path
   (m, f, n, l, r, g): a simple and elementary path

2 – Yes, the two paths (1, 5, 2, 4, 7, 3, 6) and (m, f, n, l, r, g) are identical

3 - The path (1, 5, 2, 3, 6, 5, 4, 7, 3, 5, 7, 1) is a circuit, but it is not simple, nor elementary.

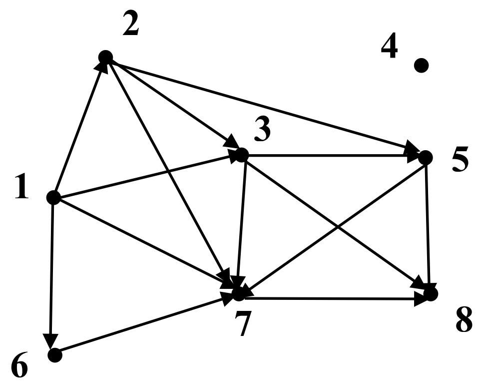
4- The path (1, 5, 2, 3, 5, 4, 7, 1): is a simple, non-elementary circuit.
The path (5, 2, 3, 4, 7, 1, 5): is a circuit, simple and elementary.

5 - (i, m, h, b, g, a): a Hamiltonian path (elementary path that passes through all the vertices of the graph)

6 – The previous path can be expressed by the following sequence of vertices:
(i, m, h, b, g, a) <==> (7, 1, 5, 4, 3, 6, 2)

Exercise 03

Let G(X,U) be a directed graph



1 – A source vertex: {1}, there is also: {4},
A well summit: {8}, there is also: {4}
Isolated peaks: {4}

2 - $d+(4) = 0, \Gamma(4) = \phi$, $d(1) = 4$, $\Gamma(1) = \{2, 3, 6, 7\}$
$d-(6) = 1$, $\Gamma^{+}(1) = \{2, 3, 6, 7\}$, $\Gamma^{-}(7) = \{1, 2, 3, 5, 6\}$,
$d(3) = 5$

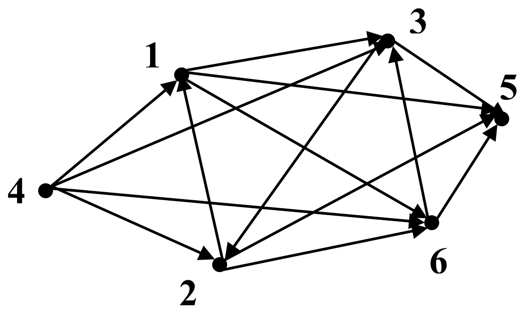
3 – The order and size of the graph G.
Order $n = |X| = 8$, Size $m = |U| = 13$

4 – A simple path between 1 and 8: (1, 3, 8), (1, 7, 8), (1, 2, 5, 7, 8}

5 – A chain between 2 and 7: (2, 1, 7), (2, 5, 3, 1, 7)

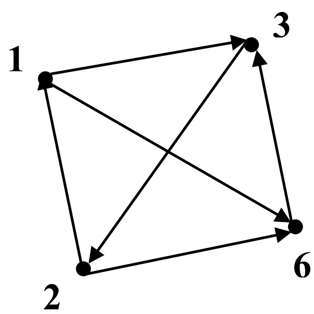
Exercise 04

Let the following graph G(X, U):

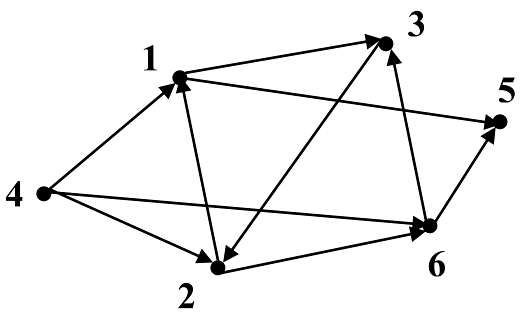


With: X = {1, 2, 3, 4, 5, 6}
U = {(4.2), (2, 6), (1.3), (6.5), (1.6), (6.3),
(2.5), (3.2), (4.1)}

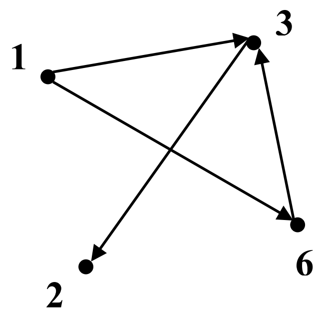
Which of the following graphs is a: subgraph, a partial graph, a partial subgraph?



(G1)



(G2)



(G3)

(G1): a subgraph of G (we have retained some vertices and some arcs)
(G2): a partial graph of G (we took all the vertices, but some arcs)
(G3): a partial subgraph of G (a partial graph of the subgraph G1)