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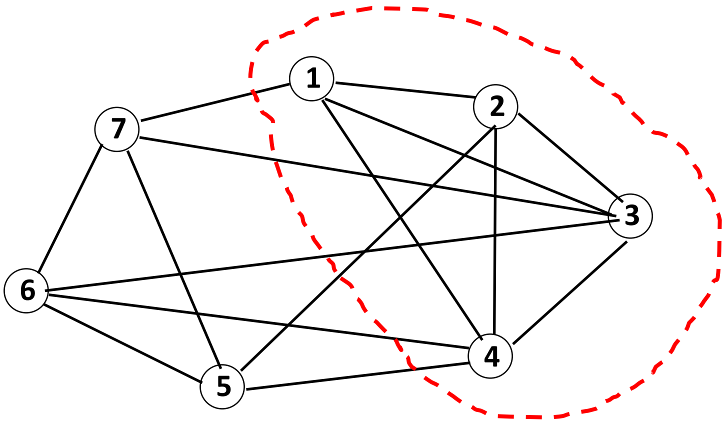
TUTORIALS - SERIES NO. 04 CORRECTED

Exercise 01 (Timetable Problem)

The courses are numbered from 1 to 7
The following courses have common students: (1, 2), (1, 3), (1, 4), (1, 7), (2, 3), (2, 4), (2, 5), (2, 7), (3, 4), (3, 6), (3, 7), (4, 5), (4, 6), (5, 6), (5, 7), (6, 7).
The graph below represents courses with common students



How to organize exams in a way that no student has taken two exams for two different courses at the same time

==> course tests having students in common must not organize at the same time.
==> a vertex coloring problem
k-coloring of G with k =γ(G)
{1, 2, 3, 4} is a complete subgraph of G,
==> a maximum clique ==>γ(G)≥4
The stable subsets of G:
S1 = {1, 6}
S2 = {2}
S3 = {3, 5}
S4 = {4, 7}

==>γ(G)≤4 ==>γ(G) = 4
The exams can be divided into 4 periods as follows:
1st period, exams for courses 1 and 6
2nd period, course 2 tests
3rd period, exams for courses 3 and 5
4th period, exams for courses 4 and 7

Solution 2:

Summit	Degree	Color
2	5	C1
3	5	C2
4	5	C3
7	5	C3
1	4	C4
5	4	C2
6	4	C1

The coloring groups are: {2, 6}, {3, 5}, {4, 7}, {1}

Exercise 02 (An aquarium problem)

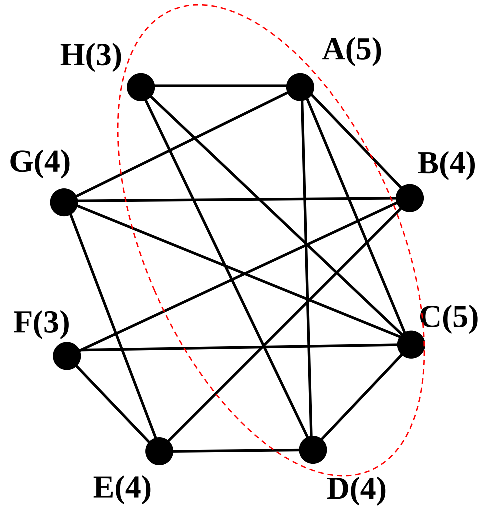
A, B, C, D, E, F, G and H denote eight fish; in the table below, a cross means that the fish cannot coexist in the same aquarium:

	A	B	C	D	E	F	G	H
A		X	X	X			X	H
B	X				X	X	X	
C	X			X		X	X	X
D	X		X		X			X
E		X		X		X	X	
F		X	X		X			
G	X	X	X		X			
H	X		X	X				

What is the minimum number of aquariums required?

We draw the incompatibility graph:

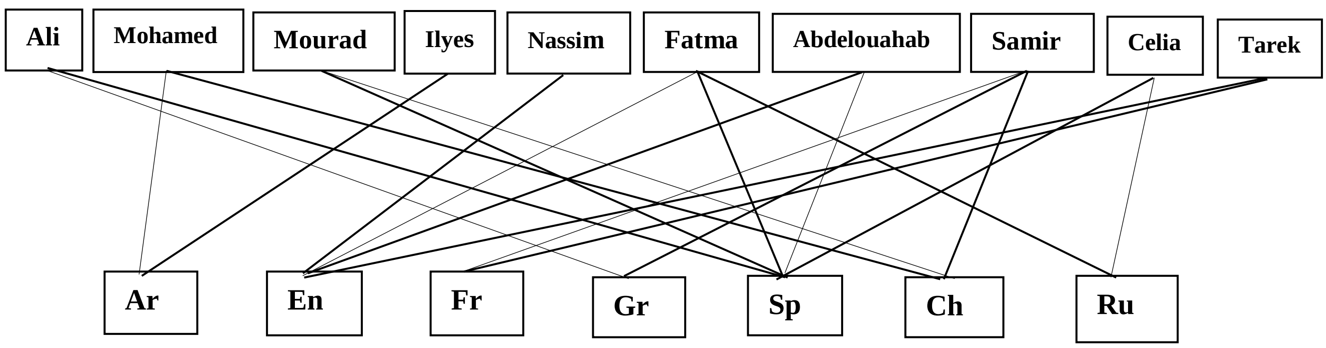
Summit	Degree	Color
A	5	C1
C	5	C2
B	4	C2
D	4	C3
E	4	C1
G	4	C3
F	3	C1
H	3	C4



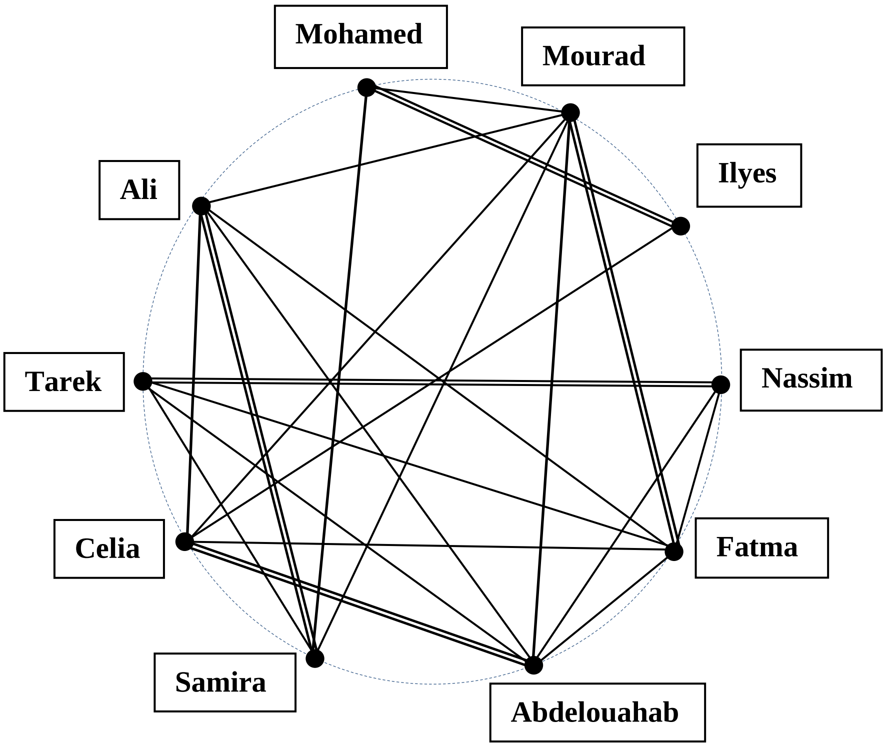
The minimum number of aquariums is 4 (= number of colors)
Aqua1: {A, E}
Aqua2: {C, B}
Aqua3: {D, G, F}
Aqua4: {H}

Exercise 03 (Coupling problem)

a) We establish the graph (person, language)



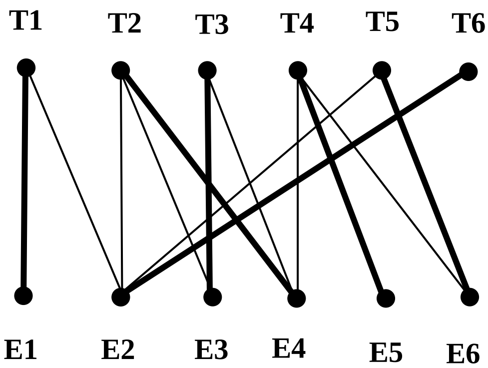
From the graph (person, language) we can draw the linguistic compatibility graph between people



We define a coupling C with |C| = 5; C is a maximum and perfect coupling.
Based on the maximum coupling edges, we draw the pairs that can communicate with each other (we take the saturated edges)
So the number of pairs is: 5

Exercise 4 (Coupling Problem):

- To maximize the number of jobs, each person can only hold one job and a job should only be held by one person
- We are looking for a maximum matching C in the graph



Max.jobs = |C| = 6

Exercise 5 (Assignment Problem):

Application of the Hungarian method

14	5	8	7	5
2	12	6	5	2
7	8	3	9	3
2	4	6	10	2

9	0	3	2
0	10	4	3
4	5	0	6
0	2	4	8
0	0	0	-2

9	0	3	0
0	10	4	1
4	5	0	4
0	2	4	6

9	$\boxed{0}$	3	$\cancel{0}$
$\boxed{0}$	10	4	1
4	5	$\boxed{0}$	4
$\cancel{0}$	2	4	6

			1	
9	0	3	0	
$\boxed{0}$	**10**	**4**	**1**	*
4	5	$\boxed{0}$	4	
$\cancel{0}$	**2**	**4**	**6**	*
*				

10	$\boxed{0}$	3	$\cancel{0}$
$\cancel{0}$	9	3	$\boxed{0}$
5	5	$\boxed{0}$	4
$\boxed{0}$	1	3	5

We see that we have a single zero framed in each row and each column ==> stop the algorithm and the optimal solution is:

14	$\boxed{5}$	8	7
2	12	6	$\boxed{5}$
7	8	$\boxed{3}$	9
$\boxed{2}$	4	6	10

The minimum total time to execute tasks on the different machines = 5+5+3+2 = 15

Good luck