

University of M'sila

Master 1 – Information Décisionnelle & Optimisation

Module: Constraint Logic Programming

Lab/Tutorial Worksheet No. 3

Exercise 1 – Arc-Consistency (AC1) on the Scheduling CSP

Given the CSP from Worksheet No. 2, Exercise 1:

- Variables: s_i (start time of task i)
- Domains: $D(s_i) = \{0, 1, \dots, 6 - p_i\}$
- Constraints: $s_1 + 1 \leq s_6$ (1), $s_2 + 1 \leq s_4$ (2), $s_3 + 2 \leq s_4$ (3), $s_4 + 1 \leq s_5$ (4), $s_5 + 2 \leq s_6$ (5), $s_1 \neq s_4$ (6)

Apply the AC1 algorithm to make this CSP arc-consistent.

Solution – Exercise 1

The arc queue Q is initialised with all ordered pairs of variables linked by a binary constraint:

$Q = \{(s_1, s_6), (s_6, s_1), (s_2, s_4), (s_4, s_2), (s_3, s_4), (s_4, s_3), (s_4, s_5), (s_5, s_4), (s_5, s_6), (s_6, s_5), (s_1, s_4), (s_4, s_1)\}$

The CSP is Node Consistent (No unary constraints).

$Ds1=\{0,1,3,4,5\}$, $Ds2=\{0,1,3,4,5\}$, $Ds3=\{0,1,3,4,5\}$, $Ds4=\{0,1,3,4,5\}$, $Ds5=\{0,1,3,4,5\}$, $Ds6=\{0,1,3,4,5\}$

AC1	Passe 01 ; R=false		R	Passe 02 ; R=false		R	Passe 03 ; R=false		R	Passe 04 ; R=false		R
Constraint	Old Domains	New Domains	F	Old D.	New D.	F	Old D.	New D.	F	Old D.	New D.	F
1	$Ds1=\{0,1,2,3,4,5\}$, $Ds6=\{0,1,2,3,4,5\}$	$Ds1=\{0,1,2,3,4\}$, $Ds6=\{1,2,3,4,5\}$	T	$Ds1=\{0,1,2,3,4\}$, $Ds6=\{5\}$	/	F	$Ds1=\{0,1,3,4\}$, $Ds6=\{5\}$	/	F	$Ds1=\{0,1,3,4\}$, $Ds6=\{5\}$	/	F
2	$Ds2=\{0,1,2,3,4,5\}$, $Ds4=\{0,1,2,3,4,5\}$	$Ds2=\{0,1,2,3,4\}$, $Ds4=\{1,2,3,4,5\}$	T	$Ds2=\{0,1,2,3,4\}$, $Ds4=\{2,3\}$	$Ds2=\{0,1,2\}$, $Ds4=\{2,3\}$	T	$Ds2=\{0,1,2\}$, $Ds4=\{2\}$	$Ds2=\{0,1\}$, $Ds4=\{2\}$	T	$Ds2=\{0,1\}$, $Ds4=\{2\}$	/	F
3	$Ds3=\{0,1,2,3,4,5\}$, $Ds4=\{1,2,3,4,5\}$	$Ds3=\{0,1,2,3\}$, $Ds4=\{2,3,4,5\}$	T	$Ds3=\{0,1,2,3\}$, $Ds4=\{2,3\}$	$Ds3=\{0,1\}$, $Ds4=\{2,3\}$	T	$Ds3=\{0,1\}$, $Ds4=\{2\}$	$Ds3=\{0\}$, $Ds4=\{2\}$	T	$Ds3=\{0\}$, $Ds4=\{2\}$	/	F
4	$Ds4=\{2,3,4,5\}$, $Ds5=\{0,1,2,3,4,5\}$	$Ds4=\{2,3\}$, $Ds5=\{3,4\}$	T	$Ds4=\{2,3\}$, $Ds5=\{3\}$	$Ds4=\{2\}$, $Ds5=\{3\}$	T	$Ds4=\{2\}$, $Ds5=\{3\}$	/	T	$Ds4=\{2\}$, $Ds5=\{3\}$	/	F
5	$Ds5=\{3,4\}$, $Ds6=\{1,2,3,4,5\}$	$Ds5=\{3\}$, $Ds6=\{5\}$	T	$Ds5=\{3\}$, $Ds6=\{5\}$	/	T	$Ds5=\{3\}$, $Ds6=\{5\}$	/	T	$Ds5=\{3\}$, $Ds6=\{5\}$	/	F
6	$Ds1=\{0,1,2,3,4\}$, $Ds4=\{2,3\}$	/	T	$Ds1=\{0,1,2,3,4\}$, $Ds4=\{2\}$	$Ds1=\{0,1,3,4\}$, $Ds4=\{2\}$	T	$Ds1=\{0,1,3,4\}$, $Ds4=\{2\}$	/	T	$Ds1=\{0,1,3,4\}$, $Ds4=\{2\}$	/	F
R	R=true		T	R=true		T	R=true		T	R=False		F

After applying AC1 (4 full passes), the resulting arc-consistent domains are:

$Ds1=\{0,1,3,4\}$, $Ds2=\{0,1\}$, $Ds3=\{0\}$, $Ds4=\{2\}$, $Ds5=\{3\}$, $Ds6=\{5\}$,