جامعة محمد بوضياف - المسيلة
Université Mohamed Boudiaf - M'sila

Mohamed Boudiaf University of M'sila
Faculty of Mathematics and Computer Science
Department of Computer Science

INTRODUCTION TO ADVANCED LINEAR PROGRAMMING

MATHEMATICAL OPTIMIZATION FOR DECISION MAKING

MANY PROBLEMS IN THE WORLD OF PRODUCTION AND CONSUMPTION CAN BE SUMMED UP AS FOLLOWS:

HOW CAN WE MAKE THE BEST USE OF RESOURCES THAT EXIST IN LIMITED QUANTITIES?

NATURALLY, ONE CANNOT ANSWER SUCH A GENERAL QUESTION PRECISELY.

First, we must express what we mean by "making the best use" - that is, we must define an objective - and we must also define the limitations of the resources or constraints that are imposed.

There are naturally many different ways to express 'goals' and constraints.

Linear programming is one of the methods that can solve such problems.

PRESENTATION OBJECTIVES

- UNDERSTAND THE FUNDAMENTAL CONCEPTS OF LINEAR PROGRAMMING.
- EXPLORE ADVANCED ASPECTS OF LINEAR PROGRAMMING.
- DISCOVER REAL-WORLD APPLICATIONS OF ADVANCED LINEAR PROGRAMMING.

WHAT IS LINEAR PROGRAMMING?

LINEAR PROGRAMMING IS A MATHEMATICAL OPTIMIZATION TECHNIQUE THAT SOLVES MAXIMIZATION OR MINIMIZATION PROBLEMS UNDER LINEAR CONSTRAINTS.

HISTORICAL

LINEAR PROGRAMMING IS A GENERAL MATHEMATICAL FRAMEWORK FOR MODELING AND SOLVING CERTAIN OPTIMIZATION PROBLEMS.

MATHEMATICALLY THE PROBLEM CONSISTS OF OPTIMIZING A LINEAR FUNCTION UNDER LINEAR CONSTRAINTS LINKING THE VARIABLES

HISTORICALLY, LINEAR PROGRAMMING WAS DEVELOPED AND USED IN 1947 BY GEORGE BERNARD DANZIG, MARSHALL WOOD AND THEIR COLLABORATORS AT THE US DEPARTMENT OF THE AIR FORCE.

THE FIRST APPLICATIONS WERE IN THE MILITARY FIELD BUT THEY QUICKLY MOVED TO INDUSTRY AND ECONOMIC PLANNING:

THIS INVOLVES, FOR EXAMPLE, DETERMINING THE PRODUCTION THAT MAXIMIZES PROFIT GIVEN LIMITED RESOURCES OR MINIMIZES COSTS WHILE GUARANTEEING A GIVEN PRODUCTION, SOLVING PROBLEMS OF ALLOCATING LIMITED RESOURCES IN ORDER TO ACHIEVE SET OBJECTIVES, ETC.

EXAMPLE OF LINEAR PROGRAMMING PROBLEM

MAXIMIZING A FIRM'S PROFITS UNDER RESOURCE CONSTRAINTS.

EXAMPLE ILLUSTRATIVE

Let's first give a concrete example: a Production Problem

- A company manufactures two products P1 and P2, each requiring the use of 3 machines M1, M2 and M3 available respectively 9900 min, 8400 min and 9600 min per week.
- A unit of product P1 uses 11 min of M1, 7 min of M2 and 6 min of M3. Similarly a unit of product P2 needs 9 min of M1, 12 min of M2 and 16 min of M3. The sales profits per unit of product are respectively 900 DA for P1 and 1000 DA for P2.

Machines	M_1	M_2	M_3	Bénéfices
P_1	11	7	6	900
P_2	9	12	16	1000
Disponibilités	9900	8400	9600	

Production problem data

One solution to this problem is to fix the production volume of these two products.

- Let x1 (resp. x2) be the number of units of P1 (resp. P2) produced; x1 and x2 are the decision variables of our issue.
- The goal is to determine the optimal weekly production, i.e. the number of manufacturing units of P1 and P2 giving the maximum profit per week taking into account the machine availability.

We must therefore seek the maximum of the economic function (or objective)

$$Max \quad F(x_1, x_2) = 900x_1 + 1000x_2$$

equal to the weekly profit corresponding to the production of a quantity x1 of product P1 and a quantity x2 of product P2 with the following constraints:

$$\begin{cases} 11x_1 + 9x_2 \leqslant 9900 \\ 7x_1 + 12x_2 \leqslant 8400 \\ 6x_1 + 16x_2 \leqslant 9600 \\ x_1 \geqslant 0, x_2 \geqslant 0 \end{cases}$$

They correspond to the manufacturing times of each product, the availability of the three machines and finally the fact that the quantities produced are positive. It is therefore a classic Linear Programming problem: it is indeed a question of determining the maximum of a linear function under linear constraints.

DECISION VARIABLES

THE UNKNOWNS WE SEEK TO OPTIMIZE.

OBJECTIVE FUNCTION

THE OBJECTIVE WE SEEK TO MAXIMIZE OR MINIMIZE.

CONSTRAINTS

THE CONDITIONS THAT LIMIT THE POSSIBLE VALUES OF DECISION VARIABLES.

MATHEMATICAL FORMULATION

EXPRESSION OF THE OBJECTIVE FUNCTION AND CONSTRAINTS.

RESOLUTION METHODS

SIMPLEX, GRADIENT METHOD, HEURISTIC METHODS, ETC.

ADVANCED LINEAR PROGRAMMING

EXPLORATION OF ADVANCED CONCEPTS SUCH AS INTEGER LINEAR PROGRAMMING, MIXED-INTEGER LINEAR PROGRAMMING, ETC.

APPLICATIONS OF ADVANCED LINEAR PROGRAMMING

CONCRETE EXAMPLES OF THE USE OF ADVANCED LINEAR PROGRAMMING IN VARIOUS FIELDS, SUCH AS LOGISTICS, FINANCE, SUPPLY CHAIN MANAGEMENT, ETC.

CONCLUSION

ADVANCED LINEAR PROGRAMMING IS A POWERFUL TOOL FOR SOLVING COMPLEX OPTIMIZATION PROBLEMS. THIS PRESENTATION WILL HELP YOU DIVE DEEPER INTO THIS EXCITING TOPIC.