

Université de M'Sila

Faculté des Mathématiques et d'Informatique / Département de Mathématiques

Master Maths. **S 1/ Analyse Mathématiques et Numériques**

– **Distributions Analyse de Fourier I** –

2025-2026

Homework (Devoir maison) December 2025

****** (Utiliser les notations du cours)

****** (On tiendra compte de la présentation des copies)

Exercise 1. Give the definition of : – a regular distribution, – a singular distribution – the norm in $\mathcal{D}(\mathbb{R}^n)$, – the derivation in $\mathcal{D}'(\mathbb{R}^n)$, – the convolution in $\mathcal{D}'(\mathbb{R})$, – the distributions $\text{pv} \frac{1}{2-3x}$, H , $\delta = \delta_{x=0}$ and $\delta_{x=1}$ in $\mathcal{D}'(\mathbb{R})$, – $L^1_{loc}(\mathbb{R})$.

Exercise 2. (1) Prove that $U \in \mathcal{D}'(\mathbb{R})$:

$$\langle U, \varphi \rangle = \lim_{\epsilon \downarrow 0} \left\{ \int_{\epsilon}^{\infty} \frac{\varphi(x)}{x^2} dx - \frac{\varphi(0)}{\epsilon} + \varphi'(0) \log \epsilon \right\}, \quad \forall \varphi \in \mathcal{D}(\mathbb{R}).$$

(2) Calculate $(x^2\delta)''$, $(x^3\delta)''$ in $\mathcal{D}'(\mathbb{R})$.

(3) Let $T(x) := (\cos x)\delta'$. Prove that $T \in \mathcal{D}'(\mathbb{R})$. Calculate T' .

Exercise 3. Let $\langle T, \varphi \rangle = \int_{-\infty}^{\infty} (1 - e^{x^3}) \varphi(x) dx$, $\forall \varphi \in \mathcal{D}(\mathbb{R})$. Prove that $T \in \mathcal{D}'(\mathbb{R}^2)$, and calculate T' in $\mathcal{D}'(\mathbb{R})$.

Exercise 4.

Let $f(x) := \frac{1}{\sqrt{\pi}} e^{-x^2}$, $x \in \mathbb{R}$.

– We put $f_j(x) := j^n f(jx)$, and $T_j := T_{f_j}$.

Calculate $\lim_{j \rightarrow \infty} T_j$ in $\mathcal{D}'(\mathbb{R}^n)$.

Exercise. Optional Let $f(x) := \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$

Calculate $f * f$, $f' * f$.

=====

29.11.2024