

Homework (Devoir maison) December 2024

** (Utiliser les notations du cours)

** (On tiendra compte de la présentation des copies)

Exercise 1. (1) Give the definition of : – a regular distribution, – a singular distribution – the norm in $\mathcal{D}(\mathbb{R}^n)$, – the derivation in $\mathcal{D}'(\mathbb{R}^n)$, – the convolution in $\mathcal{D}'(\mathbb{R})$, – the distributions $\text{pv}\frac{1}{2-x}$, H and $\delta_{x=1}$ in $\mathcal{D}'(\mathbb{R})$.

Exercise 2. Prove that $\text{pf}\frac{H}{x^2} \in \mathcal{D}'(\mathbb{R})$:

$$\langle \text{pf}\frac{H}{x^2}, \varphi \rangle = \lim_{\epsilon \downarrow 0} \left\{ \int_{\epsilon}^{\infty} \frac{\varphi(x)}{x^2} dx - \frac{\varphi(0)}{\epsilon} + \varphi'(0) \log \epsilon \right\}, \quad \forall \varphi \in \mathcal{D}(\mathbb{R}).$$

Calculate $(\text{pf}\frac{H}{x^2})'$. In the same way, define $\text{pf}\frac{H}{x^3}$ in $\mathcal{D}'(\mathbb{R})$ and calculate $(\text{pf}\frac{H}{x^3})'$.

Exercise 3. Let $T(x) := (\sin x)\delta''$. Prove that $T \in \mathcal{D}'(\mathbb{R})$. Calculate T' .

Exercise 4. Let

$$\langle T, \varphi \rangle = \int_{-\infty}^{\infty} \varphi(2x+3, -x) dx, \quad \forall \varphi \in \mathcal{D}(\mathbb{R}^2).$$

Prove that $T \in \mathcal{D}'(\mathbb{R}^2)$, and calculate $(\partial_x - \partial_y)T$ in $\mathcal{D}'(\mathbb{R}^2)$.

Exercise 5. Let $f(x) := \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$

Calculate $f * f$, $f' * f''$.

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