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> #TP03
> #EXO 01
> restart;
> with(LinearAlgebra):
> A:=VandermondeMatrix(<a,b,c,d,e>);
> Determinant(A);
> factor(Determinant(A));
      
$$-(b-d)(c-d)(c-b)(-d+a)(a-b)(a-c)(-d+e)(e-b)(e-c)(e-a)$$


> #exo02
> restart;
> with(LinearAlgebra):
> A:=<<a , -b, -c, -d>| <b, a, d, -c>| <c, -d, a, b>|<d, c, -b, a>>>;
> A.Transpose(A);
> Determinant(A);
> factor(%);
> CharacteristicPolynomial(A,x);
> factor(%);
> Delta:=JordanForm(A);

> #exo 03
> restart;
> with(LinearAlgebra):
> A:=<<58 , -29, -145>| <52, 187, 65>| <36, 9, 219>>>;
> A^(-1);
> Delta:=JordanForm(A);
> P:=JordanForm(A,output='Q');
> A:=P.Delta.P^(-1);
> puissance:=x->x^n;
> Deltan:=map(puissance, Delta);
> An:=P.Deltan.P^(-1);
> subs(n=0,An);
> subs(n=1,An);
> subs(n=2,An);
> subs(n=5,An);
> A^5;

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> #EXO 04
> restart; with (LinearAlgebra) :
> A:=Matrix ([[ -1, 4, 3] , [2, 1, 2 ] , [ 3, 2, 0]]) ;
> B:=Matrix ([[1, 0, -3] , [-5, 2, -2 ] , [ -6, 2, 1]]) ;

> NullSpace(A) ; ColumnSpace(A) ; NullSpace(B) ;
ColumnSpace(B) ;

> M:= A+lambda*B ;
> S:=Determinant(M) ;
> solve( S,{lambda} ) ;
> a:=Vector([1, 1, 1]); LinearSolve( A,a);
LinearSolve(B,a);
Error, (in LinearAlgebra:-LA_Main:-LinearSolve)
inconsistent system
> b:=Vector([1, 1, 0]); LinearSolve( A,b);
LinearSolve(B,b);

> #EXO 05
> restart; with (LinearAlgebra) :
> A:=GenerateMatrix([ x - y -z - 3* t, y + 3* z + t, -x +
y+ z + 3* t, -3* x + y + 3*z - 2* t] , [x, y, z, t])[1] ;
> Rank(A) ; F:=ColumnSpace(A) ;
> G:=NullSpace(A) ;
> SumBasis([F, G]) ; IntersectionBasis([F, G]) ;

> #EXO 6
>
> # 1
> restart;
> with(LinearAlgebra):
> f:=(i,j)->abs(i-j);

> g:=(i,j)->i+j;
> Matrix(4,4,f); Matrix(4,4,g);
>
for n from 2 to 10 do
n, Rank(Matrix(n,n,f)), Rank(Matrix(n,n,g));

end do;

>for n from 10 to 100 while n =Rank(Matrix(n,n,f)) and 2
= Rank(Matrix(n,n,g)) do
end do:
n;

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